

3.3 Function Notation

Essential Question How can you use function notation to represent a function?

The notation $f(x)$, called **function notation**, is another name for y . This notation is read as “the value of f at x ” or “ f of x .” The parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g , h , j , and k are often used to name functions.

EXPLORATION 1 Matching Functions with Their Graphs

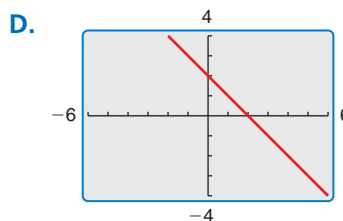
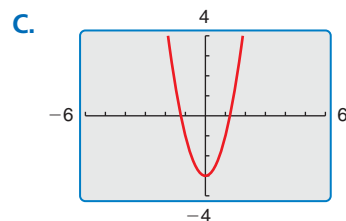
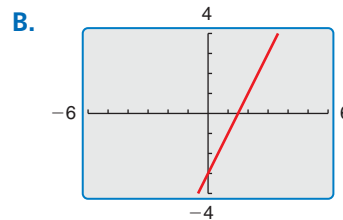
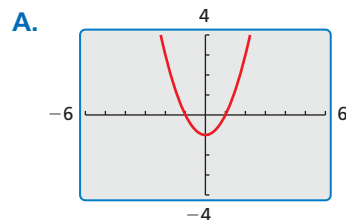
Work with a partner. Match each function with its graph.

a. $f(x) = 2x - 3$

b. $g(x) = -x + 2$

c. $h(x) = x^2 - 1$

d. $j(x) = 2x^2 - 3$



ATTENDING TO PRECISION

To be proficient in math, you need to use clear definitions and state the meanings of the symbols you use.

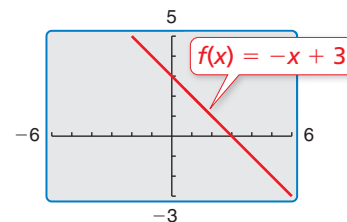
EXPLORATION 2 Evaluating a Function

Work with a partner. Consider the function

$$f(x) = -x + 3$$

Locate the points $(x, f(x))$ on the graph. Explain how you found each point.

- a. $(-1, f(-1))$
- b. $(0, f(0))$
- c. $(1, f(1))$
- d. $(2, f(2))$



Communicate Your Answer

3. How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

Standard Notation

$$y = 2x + 5$$

Function Notation

$$f(x) = 2x + 5$$

3.3 Lesson

Core Vocabulary

function notation, p. 122

Previous

linear function
quadrant

READING

The notation $f(x)$ is read as “the value of f at x ” or “ f of x .” It does not mean “ f times x .”

What You Will Learn

- ▶ Use function notation to evaluate and interpret functions.
- ▶ Use function notation to solve and graph functions.
- ▶ Solve real-life problems using function notation.

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form $y = mx + b$. By naming a linear function f , you can also write the function using **function notation**.

$$f(x) = mx + b \quad \text{Function notation}$$

The notation $f(x)$ is another name for y . If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x . You can use letters other than f to name a function, such as g or h .

EXAMPLE 1 Evaluating a Function

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x .	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

- ▶ When $x = 2$, $f(x) = -1$, and when $x = -2$, $f(x) = 15$.

EXAMPLE 2 Interpreting Function Notation

Let $f(t)$ be the outside temperature ($^{\circ}\text{F}$) t hours after 6 A.M. Explain the meaning of each statement.

- a. $f(0) = 58$ b. $f(6) = n$ c. $f(3) < f(9)$

SOLUTION

- a. The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F .
- b. The output of f when $t = 6$ is n . So, the temperature at noon (6 hours after 6 A.M.) is $n^{\circ}\text{F}$.
- c. The output of f when $t = 3$ is less than the output of f when $t = 9$. So, the temperature at 9 A.M. (3 hours after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

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Evaluate the function when $x = -4, 0$, and 3 .

1. $f(x) = 2x - 5$ 2. $g(x) = -x - 1$
3. **WHAT IF?** In Example 2, let $f(t)$ be the outside temperature ($^{\circ}\text{F}$) t hours after 9 A.M. Explain the meaning of each statement.
- a. $f(4) = 75$ b. $f(m) = 70$ c. $f(2) = f(9)$ d. $f(6) > f(0)$

Using Function Notation to Solve and Graph

EXAMPLE 3 Solving for the Independent Variable

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

SOLUTION

$$h(x) = \frac{2}{3}x - 5$$

Write the function.

$$-7 = \frac{2}{3}x - 5$$

Substitute -7 for $h(x)$.

$$\underline{+5} \quad \underline{+5}$$

Add 5 to each side.

$$-2 = \frac{2}{3}x$$

Simplify.

$$\frac{3}{2} \cdot (-2) = \frac{3}{2} \cdot \frac{2}{3}x$$

Multiply each side by $\frac{3}{2}$.

$$-3 = x$$

Simplify.

► When $x = -3$, $h(x) = -7$.

EXAMPLE 4 Graphing a Linear Function in Function Notation

Graph $f(x) = 2x + 5$.

SOLUTION

Step 1 Make an input-output table to find ordered pairs.

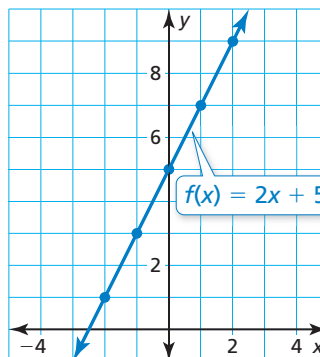
x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.

STUDY TIP

The graph of $y = f(x)$ consists of the points $(x, f(x))$.



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Find the value of x so that the function has the given value.

4. $f(x) = 6x + 9$; $f(x) = 21$

5. $g(x) = -\frac{1}{2}x + 3$; $g(x) = -1$

Graph the linear function.

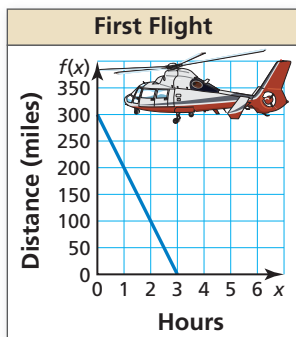
6. $f(x) = 3x - 2$

7. $g(x) = -x + 4$

8. $h(x) = -\frac{3}{4}x - 1$

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics



The graph shows the number of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

SOLUTION

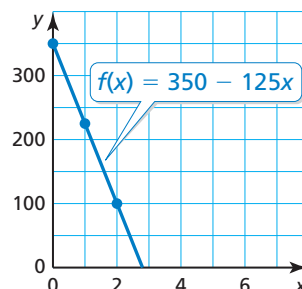
- 1. Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.
- 2. Make a Plan** Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The x -value that corresponds to $f(x) = 0$ represents the flight time.
- 3. Solve the Problem** Graph $f(x) = 350 - 125x$.

Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	-25

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. Note that the function only makes sense when x and $f(x)$ are positive. So, only draw the line in the first quadrant.



► From the graph of the first flight, you can see that when $f(x) = 0$, $x = 3$. From the graph of the second flight, you can see that when $f(x) = 0$, x is slightly less than 3. So, the second flight takes less time.

- 4. Look Back** You can check that your answer is correct by finding the value of x for which $f(x) = 0$.

$$f(x) = 350 - 125x \quad \text{Write the function.}$$

$$0 = 350 - 125x \quad \text{Substitute 0 for } f(x).$$

$$-350 = -125x \quad \text{Subtract 350 from each side.}$$

$$2.8 = x \quad \text{Divide each side by } -125.$$

So, the second flight takes 2.8 hours, which is less than 3.

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- 9. WHAT IF?** Let $f(x) = 250 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

3.3 Exercises

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** When you write the function $y = 2x + 10$ as $f(x) = 2x + 10$, you are using _____.
- REASONING** Your height can be represented by a function h , where the input is your age. What does $h(14)$ represent?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, evaluate the function when $x = -2, 0$, and 5. (See Example 1.)

- | | |
|-------------------------|-------------------------|
| 3. $f(x) = x + 6$ | 4. $g(x) = 3x$ |
| 5. $h(x) = -2x + 9$ | 6. $r(x) = -x - 7$ |
| 7. $p(x) = -3 + 4x$ | 8. $b(x) = 18 - 0.5x$ |
| 9. $v(x) = 12 - 2x - 5$ | 10. $n(x) = -1 - x + 4$ |

11. **INTERPRETING FUNCTION NOTATION** Let $c(t)$ be the number of customers in a restaurant t hours after 8 A.M. Explain the meaning of each statement. (See Example 2.)

- | | |
|----------------|--------------------|
| a. $c(0) = 0$ | b. $c(3) = c(8)$ |
| c. $c(n) = 29$ | d. $c(13) < c(12)$ |

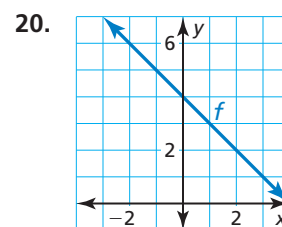
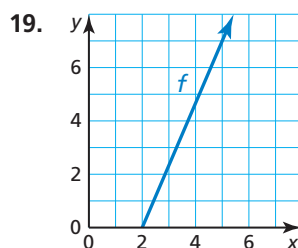
12. **INTERPRETING FUNCTION NOTATION** Let $H(x)$ be the percent of U.S. households with Internet use x years after 1980. Explain the meaning of each statement.

- | | |
|----------------------------------|---------------|
| a. $H(23) = 55$ | b. $H(4) = k$ |
| c. $H(27) \geq 61$ | |
| d. $H(17) + H(21) \approx H(29)$ | |

In Exercises 13–18, find the value of x so that the function has the given value. (See Example 3.)

- $h(x) = -7x; h(x) = 63$
- $t(x) = 3x; t(x) = 24$
- $m(x) = 4x + 15; m(x) = 7$
- $k(x) = 6x - 12; k(x) = 18$
- $q(x) = \frac{1}{2}x - 3; q(x) = -4$
- $j(x) = -\frac{4}{5}x + 7; j(x) = -5$

In Exercises 19 and 20, find the value of x so that $f(x) = 7$.



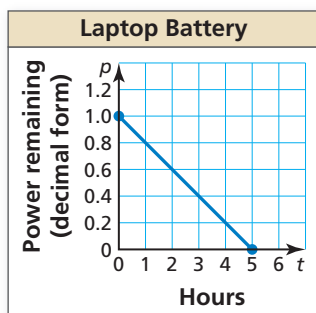
21. **MODELING WITH MATHEMATICS** The function $C(x) = 17.5x - 10$ represents the cost (in dollars) of buying x tickets to the orchestra with a \$10 coupon.
- How much does it cost to buy five tickets?
 - How many tickets can you buy with \$130?
22. **MODELING WITH MATHEMATICS** The function $d(t) = 300,000t$ represents the distance (in kilometers) that light travels in t seconds.
- How far does light travel in 15 seconds?
 - How long does it take light to travel 12 million kilometers?



In Exercises 23–28, graph the linear function. (See Example 4.)

- | | |
|--------------------------------|-------------------------------|
| 23. $p(x) = 4x$ | 24. $h(x) = -5$ |
| 25. $d(x) = -\frac{1}{2}x - 3$ | 26. $w(x) = \frac{3}{5}x + 2$ |
| 27. $g(x) = -4 + 7x$ | 28. $f(x) = 3 - 6x$ |

29. **PROBLEM SOLVING** The graph shows the percent p (in decimal form) of battery power remaining in a laptop computer after t hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer's battery will last longer? Explain. (See Example 5.)



30. **PROBLEM SOLVING** The function $C(x) = 25x + 50$ represents the labor cost (in dollars) for Certified Remodeling to build a deck, where x is the number of hours of labor. The table shows sample labor costs from its main competitor, Master Remodeling. The deck is estimated to take 8 hours of labor. Which company would you hire? Explain.

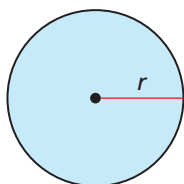
Hours	Cost
2	\$130
4	\$160
6	\$190

31. **MAKING AN ARGUMENT** Let $P(x)$ be the number of people in the U.S. who own a cell phone x years after 1990. Your friend says that $P(x + 1) > P(x)$ for any x because $x + 1$ is always greater than x . Is your friend correct? Explain.

32. **THOUGHT PROVOKING** Let $B(t)$ be your bank account balance after t days. Describe a situation in which $B(0) < B(4) < B(2)$.

33. **MATHEMATICAL CONNECTIONS** Rewrite each geometry formula using function notation. Evaluate each function when $r = 5$ feet. Then explain the meaning of the result.

- Diameter, $d = 2r$
- Area, $A = \pi r^2$
- Circumference, $C = 2\pi r$



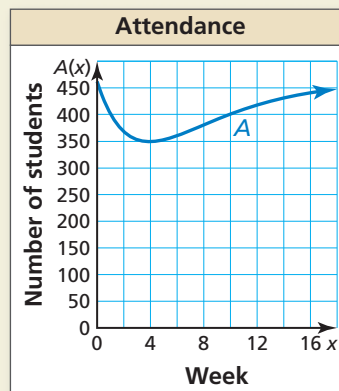
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. (Section 2.5)

- $-2 \leq x - 11 \leq 6$
- $5a < -35$ or $a - 14 > 1$
- $-16 < 6k + 2 < 0$
- $2d + 7 < -9$ or $4d - 1 > -3$
- $5 \leq 3y + 8 < 17$
- $4v + 9 \leq 5$ or $-3v \geq -6$

34. **HOW DO YOU SEE IT?** The function $y = A(x)$ represents the attendance at a high school x weeks after a flu outbreak. The graph of the function is shown.



- What happens to the school's attendance after the flu outbreak?
- Estimate $A(13)$ and explain its meaning.
- Use the graph to estimate the solution(s) of the equation $A(x) = 400$. Explain the meaning of the solution(s).
- What was the least attendance? When did that occur?
- How many students do you think are enrolled at this high school? Explain your reasoning.

35. **INTERPRETING FUNCTION NOTATION** Let f be a function. Use each statement to find the coordinates of a point on the graph of f .

- $f(5)$ is equal to 9.
- A solution of the equation $f(n) = -3$ is 5.

36. **REASONING** Given a function f , tell whether the statement

$$f(a + b) = f(a) + f(b)$$

is true or false for all inputs a and b . If it is false, explain why.