5.1 Solving Systems of Linear Equations by Graphing

Essential Question How can you solve a system of linear equations?

EXPLORATION 1 Writing a System of Linear Equations

Work with a partner. Your family opens a bed-and-breakfast. They spend \$600 preparing a bedroom to rent. The cost to your family for food and utilities is \$15 per night. They charge \$75 per night to rent the bedroom.

a. Write an equation that represents the costs.

$$\underbrace{\text{Cost, } C \text{ (in dollars)}}_{} = \underbrace{\$15 \text{ per night}}_{} \cdot \underbrace{\text{Number of nights, } x}_{} + \underbrace{\$600}_{}$$

b. Write an equation that represents the revenue (income).

$$\underbrace{\text{Revenue, } R \text{ (in dollars)}}_{} = \underbrace{\$75 \text{ per night}}_{} \cdot \underbrace{\text{Number of nights, } x}_{}$$

MODELING WITH MATHEMATICS

To be proficient in math, you need to identify important quantities in real-life problems and map their relationships using tools such as diagrams, tables, and graphs.

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

EXPLORATION 2 Using a Table or Graph to Solve a System

Work with a partner. Use the cost and revenue equations from Exploration 1 to determine how many nights your family needs to rent the bedroom before recovering the cost of preparing the bedroom. This is the *break-even point*.

a. Copy and complete the table.

x (nights)	0	1	2	3	4	5	6	7	8	9	10	11
C (dollars)												
R (dollars)												

b. How many nights does your family need to rent the bedroom before breaking even?

c. In the same coordinate plane, graph the cost equation and the revenue equation from Exploration 1.

d. Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in part (b)? Explain.

Communicate Your Answer

3. How can you solve a system of linear equations? How can you check your solution?

4. Solve each system by using a table or sketching a graph. Explain why you chose each method. Use a graphing calculator to check each solution.

a. $y = -4.3x - 1.3$
$y = 1.7x + 4.7$

b. $y = x$
$y = -3x + 8$

c. $y = -x - 1$
$y = 3x + 5$

5.1 Lesson

What You Will Learn

- Check solutions of systems of linear equations.
- Solve systems of linear equations by graphing.
- Use systems of linear equations to solve real-life problems.

Core Vocabulary

system of linear equations, *p. 236*

solution of a system of linear equations, *p. 236*

Previous

linear equation

ordered pair

Systems of Linear Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$x + y = 7$ Equation 1

$2x - 3y = -11$ Equation 2

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the system of linear equations.

a. $(2, 5)$; $x + y = 7$ Equation 1, $2x - 3y = -11$ Equation 2

b. $(-2, 0)$; $y = -2x - 4$ Equation 1, $y = x + 4$ Equation 2

SOLUTION

a. Substitute 2 for x and 5 for y in each equation.

Equation 1

$x + y = 7$

$2 + 5 \stackrel{?}{=} 7$

$7 = 7$ ✓

Equation 2

$2x - 3y = -11$

$2(2) - 3(5) \stackrel{?}{=} -11$

$-11 = -11$ ✓

READING

A system of linear equations is also called a *linear system*.

▶ Because the ordered pair $(2, 5)$ is a solution of each equation, it is a solution of the linear system.

b. Substitute -2 for x and 0 for y in each equation.

Equation 1

$y = -2x - 4$

$0 \stackrel{?}{=} -2(-2) - 4$

$0 = 0$ ✓

Equation 2

$y = x + 4$

$0 \stackrel{?}{=} -2 + 4$

$0 \neq 2$ ✗

▶ The ordered pair $(-2, 0)$ is a solution of the first equation, but it is not a solution of the second equation. So, $(-2, 0)$ is *not* a solution of the linear system.

Monitoring Progress

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Tell whether the ordered pair is a solution of the system of linear equations.

1. $(1, -2)$; $2x + y = 0$, $-x + 2y = 5$

2. $(1, 4)$; $y = 3x + 1$, $y = -x + 5$

Solving Systems of Linear Equations by Graphing

The solution of a system of linear equations is the point of intersection of the graphs of the equations.

Core Concept

Solving a System of Linear Equations by Graphing

Step 1 Graph each equation in the same coordinate plane.

Step 2 Estimate the point of intersection.

Step 3 Check the point from Step 2 by substituting for x and y in each equation of the original system.

REMEMBER

Note that the linear equations are in slope-intercept form. You can use the method presented in Section 3.5 to graph the equations.

EXAMPLE 2 Solving a System of Linear Equations by Graphing

Solve the system of linear equations by graphing.

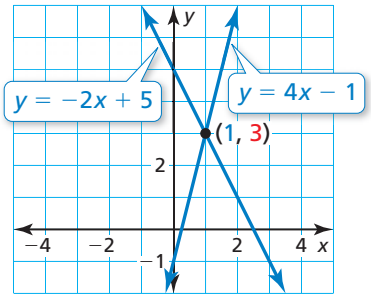
$y = -2x + 5$ Equation 1

$y = 4x - 1$ Equation 2

SOLUTION

Step 1 Graph each equation.

Step 2 Estimate the point of intersection.
The graphs appear to intersect at (1, 3).



Step 3 Check your point from Step 2.

Equation 1

$y = -2x + 5$

$3 \stackrel{?}{=} -2(1) + 5$

$3 = 3$ ✓

Equation 2

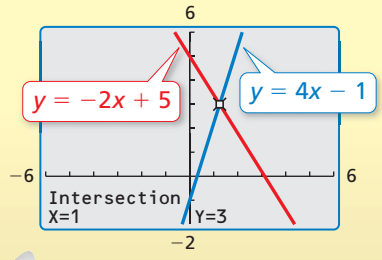
$y = 4x - 1$

$3 \stackrel{?}{=} 4(1) - 1$

$3 = 3$ ✓

▶ The solution is (1, 3).

Check



Monitoring Progress

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Solve the system of linear equations by graphing.

3. $y = x - 2$
$y = -x + 4$

4. $y = \frac{1}{2}x + 3$
$y = -\frac{3}{2}x - 5$

5. $2x + y = 5$
$3x - 2y = 4$

Solving Real-Life Problems

EXAMPLE 3 Modeling with Mathematics

A roofing contractor buys 30 bundles of shingles and 4 rolls of roofing paper for \$1040. In a second purchase (at the same prices), the contractor buys 8 bundles of shingles for \$256. Find the price per bundle of shingles and the price per roll of roofing paper.

SOLUTION

1. **Understand the Problem** You know the total price of each purchase and how many of each item were purchased. You are asked to find the price of each item.
2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Words

$$30 \cdot \boxed{\text{Price per bundle}} + 4 \cdot \boxed{\text{Price per roll}} = 1040$$

$$8 \cdot \boxed{\text{Price per bundle}} + 0 \cdot \boxed{\text{Price per roll}} = 256$$

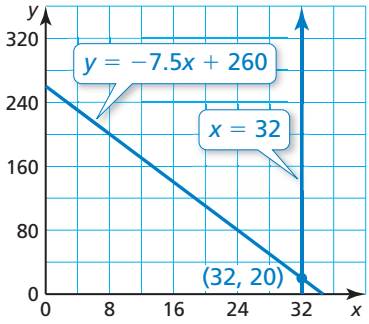
Variables Let x be the price (in dollars) per bundle and let y be the price (in dollars) per roll.

System

$$30x + 4y = 1040 \quad \text{Equation 1}$$

$$8x = 256 \quad \text{Equation 2}$$

Step 1 Graph each equation. Note that only the first quadrant is shown because x and y must be positive.



Step 2 Estimate the point of intersection. The graphs appear to intersect at (32, 20).

Step 3 Check your point from Step 2.

Equation 1

$$30x + 4y = 1040$$
$$30(32) + 4(20) \stackrel{?}{=} 1040$$
$$1040 = 1040 \checkmark$$

Equation 2

$$8x = 256$$
$$8(32) \stackrel{?}{=} 256$$
$$256 = 256 \checkmark$$

▶ The solution is (32, 20). So, the price per bundle of shingles is \$32, and the price per roll of roofing paper is \$20.

4. **Look Back** You can use estimation to check that your solution is reasonable. A bundle of shingles costs about \$30. So, 30 bundles of shingles and 4 rolls of roofing paper (at \$20 per roll) cost about 30(30) + 4(20) = \$980, and 8 bundles of shingles costs about 8(30) = \$240. These prices are close to the given values, so the solution seems reasonable.

Monitoring Progress

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6. You have a total of 18 math and science exercises for homework. You have six more math exercises than science exercises. How many exercises do you have in each subject?

5.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Do the equations $5y - 2x = 18$ and $6x = -4y - 10$ form a system of linear equations? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the system of linear equations $-4x + 2y = 4$ and $4x - y = -6$. Which is different? Find "both" answers.

Solve the system of linear equations.

Solve each equation for y.

Find the point of intersection of the graphs of the equations.

Find an ordered pair that is a solution of each equation in the system.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell whether the ordered pair is a solution of the system of linear equations. *(See Example 1.)*

3. $(2, 6)$; $\begin{aligned} x + y &= 8 \\ 3x - y &= 0 \end{aligned}$

4. $(8, 2)$; $\begin{aligned} x - y &= 6 \\ 2x - 10y &= 4 \end{aligned}$

5. $(-1, 3)$; $\begin{aligned} y &= -7x - 4 \\ y &= 8x + 5 \end{aligned}$

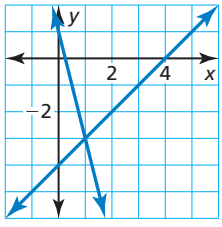
6. $(-4, -2)$; $\begin{aligned} y &= 2x + 6 \\ y &= -3x - 14 \end{aligned}$

7. $(-2, 1)$; $\begin{aligned} 6x + 5y &= -7 \\ 2x - 4y &= -8 \end{aligned}$

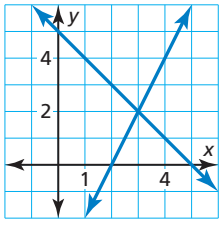
8. $(5, -6)$; $\begin{aligned} 6x + 3y &= 12 \\ 4x + y &= 14 \end{aligned}$

In Exercises 9–12, use the graph to solve the system of linear equations. Check your solution.

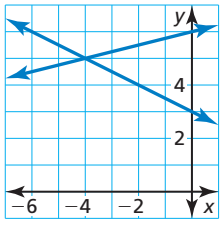
9. $\begin{aligned} x - y &= 4 \\ 4x + y &= 1 \end{aligned}$



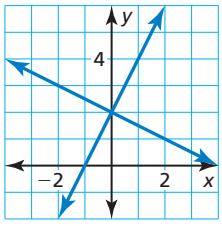
10. $\begin{aligned} x + y &= 5 \\ y - 2x &= -4 \end{aligned}$



11. $\begin{aligned} 6y + 3x &= 18 \\ -x + 4y &= 24 \end{aligned}$



12. $\begin{aligned} 2x - y &= -2 \\ 2x + 4y &= 8 \end{aligned}$



In Exercises 13–20, solve the system of linear equations by graphing. *(See Example 2.)*

13. $\begin{aligned} y &= -x + 7 \\ y &= x + 1 \end{aligned}$

14. $\begin{aligned} y &= -x + 4 \\ y &= 2x - 8 \end{aligned}$

15. $\begin{aligned} y &= \tfrac{1}{3}x + 2 \\ y &= \tfrac{2}{3}x + 5 \end{aligned}$

16. $\begin{aligned} y &= \tfrac{3}{4}x - 4 \\ y &= -\tfrac{1}{2}x + 11 \end{aligned}$

17. $\begin{aligned} 9x + 3y &= -3 \\ 2x - y &= -4 \end{aligned}$

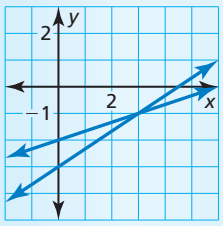
18. $\begin{aligned} 4x - 4y &= 20 \\ y &= -5 \end{aligned}$

19. $\begin{aligned} x - 4y &= -4 \\ -3x - 4y &= 12 \end{aligned}$

20. $\begin{aligned} 3y + 4x &= 3 \\ x + 3y &= -6 \end{aligned}$

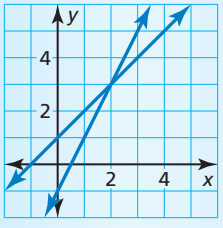
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the system of linear equations.**

21.



The solution of the linear system $x - 3y = 6$ and $2x - 3y = 3$ is $(3, -1)$.

22.



The solution of the linear system $y = 2x - 1$ and $y = x + 1$ is $x = 2$.

USING TOOLS **In Exercises 23–26, use a graphing calculator to solve the system of linear equations.**

23. $0.2x + 0.4y = 4$
$-0.6x + 0.6y = -3$

24. $-1.6x - 3.2y = -24$
$2.6x + 2.6y = 26$

25. $-7x + 6y = 0$
$0.5x + y = 2$

26. $4x - y = 1.5$
$2x + y = 1.5$

27. **MODELING WITH MATHEMATICS** You have 40 minutes to exercise at the gym, and you want to burn 300 calories total using both machines. How much time should you spend on each machine? *(See Example 3.)*

Elliptical Trainer

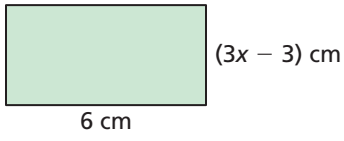
8 calories per minute

Stationary Bike

6 calories per minute

28. **MODELING WITH MATHEMATICS** You sell small and large candles at a craft fair. You collect \$144 selling a total of 28 candles. How many of each type of candle did you sell?

29. **MATHEMATICAL CONNECTIONS** Write a linear equation that represents the area and a linear equation that represents the perimeter of the rectangle. Solve the system of linear equations by graphing. Interpret your solution.



30. **THOUGHT PROVOKING** Your friend's bank account balance (in dollars) is represented by the equation $y = 25x + 250$, where x is the number of months. Graph this equation. After 6 months, you want to have the same account balance as your friend. Write a linear equation that represents your account balance. Interpret the slope and y-intercept of the line that represents your account balance.

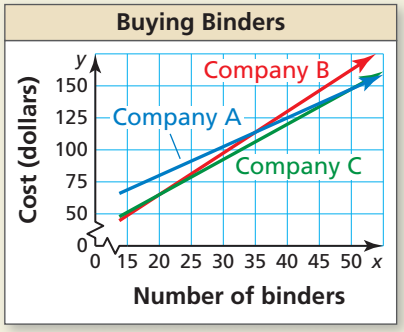
31. **COMPARING METHODS** Consider the equation $x + 2 = 3x - 4$.

a. Solve the equation using algebra.

b. Solve the system of linear equations $y = x + 2$ and $y = 3x - 4$ by graphing.

c. How is the linear system and the solution in part (b) related to the original equation and the solution in part (a)?

32. **HOW DO YOU SEE IT?** A teacher is purchasing binders for students. The graph shows the total costs of ordering x binders from three different companies.



a. For what numbers of binders are the costs the same at two different companies? Explain.

b. How do your answers in part (a) relate to systems of linear equations?

33. **MAKING AN ARGUMENT** You and a friend are going hiking but start at different locations. You start at the trailhead and walk 5 miles per hour. Your friend starts 3 miles from the trailhead and walks 3 miles per hour.

a. Write and graph a system of linear equations that represents this situation.

b. Your friend says that after an hour of hiking you will both be at the same location on the trail. Is your friend correct? Use the graph from part (a) to explain your answer.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the literal equation for y. *(Section 1.5)*

34. $10x + 5y = 5x + 20$

35. $9x + 18 = 6y - 3x$

36. $\frac{3}{4}x + \frac{1}{4}y = 5$