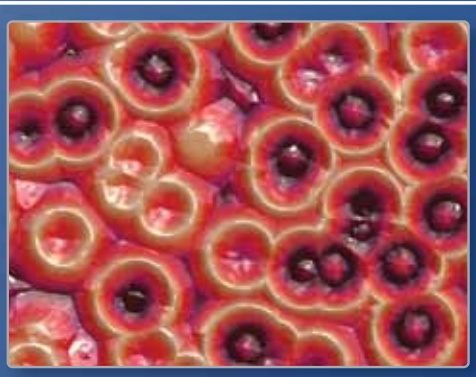
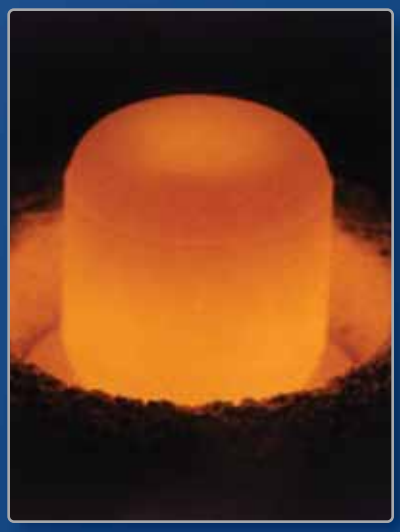
6 Exponential Functions and Sequences

Fibonacci and Flowers *(p. 343)*

Soup Kitchen *(p. 338)*



Bacterial Culture *(p. 330)*



Plutonium Decay *(p. 321)*

Coyote Population *(p. 311)*

Maintaining Mathematical Proficiency

Using Order of Operations

Example 1 **Evaluate $10^2 \div (30 \div 3) - 4(3 - 9) + 5^1$.**

First: Parentheses $10^2 \div (30 \div 3) - 4(3 - 9) + 5^1 = 10^2 \div 10 - 4(-6) + 5^1$

Second: Exponents $= 100 \div 10 - 4(-6) + 5$

Third: Multiplication and Division (from left to right) $= 10 + 24 + 5$

Fourth: Addition and Subtraction (from left to right) $= 39$

Evaluate the expression.

1. $12\left(\frac{14}{2}\right) - 3^3 + 15 - 9^2$ **2.** $5^2 \cdot 8 \div 2^2 + 20 \cdot 3 - 4$ **3.** $-7 + 16 \div 2^4 + (10 - 4^2)$

Finding Square Roots

Example 2 **Find $-\sqrt{81}$.**

▶ $-\sqrt{81}$ represents the negative square root. Because $9^2 = 81$, $-\sqrt{81} = -\sqrt{9^2} = -9$.

Find the square root(s).

4. $\sqrt{64}$ **5.** $-\sqrt{4}$ **6.** $-\sqrt{25}$ **7.** $\pm\sqrt{121}$

Writing Equations for Arithmetic Sequences

Example 3 **Write an equation for the *n*th term of the arithmetic sequence 5, 15, 25, 35, ...**

The first term is 5, and the common difference is 10.

$a_n = a_1 + (n - 1)d$ Equation for an arithmetic sequence

$a_n = 5 + (n - 1)(10)$ Substitute 5 for a_1 and 10 for d.

$a_n = 10n - 5$ Simplify.

Write an equation for the *n*th term of the arithmetic sequence.

8. 12, 14, 16, 18, . . . **9.** 6, 3, 0, −3, . . . **10.** 22, 15, 8, 1, . . .

11. **ABSTRACT REASONING** Recall that a perfect square is a number with integers as its square roots. Is the product of two perfect squares always a perfect square? Is the quotient of two perfect squares always a perfect square? Explain your reasoning.

Mathematical Practices

Mathematically proficient students look closely to find a pattern.

Problem-Solving Strategies

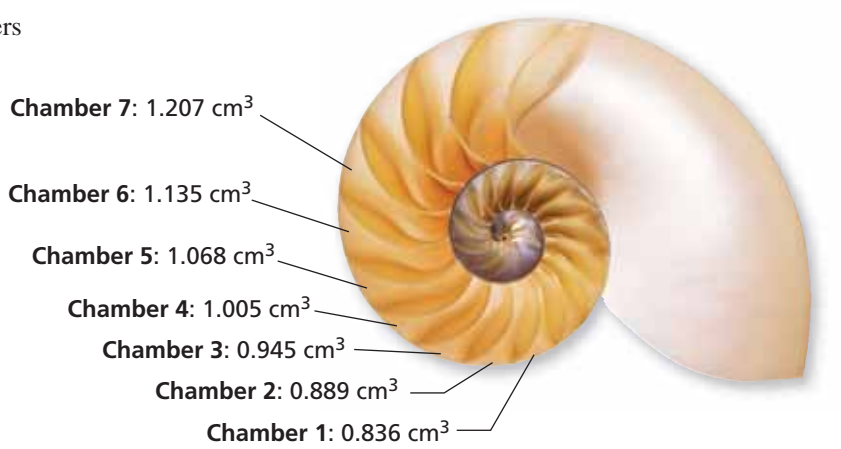
Core Concept

Finding a Pattern

When solving a real-life problem, look for a pattern in the data. The pattern could include repeating items, numbers, or events. After you find the pattern, describe it and use it to solve the problem.

EXAMPLE 1 Using a Problem-Solving Strategy

The volumes of seven chambers of a chambered nautilus are given. Find the volume of Chamber 10.



SOLUTION

To find a pattern, try dividing each volume by the volume of the previous chamber.

$$\frac{0.889}{0.836} \approx 1.063 \qquad \frac{0.945}{0.889} \approx 1.063 \qquad \frac{1.005}{0.945} \approx 1.063$$

$$\frac{1.068}{1.005} \approx 1.063 \qquad \frac{1.135}{1.068} \approx 1.063 \qquad \frac{1.207}{1.135} \approx 1.063$$

From this, you can see that the volume of each chamber is about 6.3% greater than the volume of the previous chamber. To find the volume of Chamber 10, multiply the volume of Chamber 7 by 1.063 three times.

$1.207(1.063) \approx \mathbf{1.283}$ ← volume of Chamber 8

$1.283(1.063) \approx \mathbf{1.364}$ ← volume of Chamber 9

$1.364(1.063) \approx \mathbf{1.450}$ ← volume of Chamber 10

▶ The volume of Chamber 10 is about 1.450 cubic centimeters.

Monitoring Progress

1. A rabbit population over 8 consecutive years is given by 50, 80, 128, 205, 328, 524, 839, 1342. Find the population in the tenth year.
2. The sums of the numbers in the first eight rows of Pascal's Triangle are 1, 2, 4, 8, 16, 32, 64, 128. Find the sum of the numbers in the tenth row.