# 6 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

## 6.1 Properties of Exponents

(pp. 291–298)

Simplify $\left(\frac{x}{4}\right)^{-4}$. Write your answer using only positive exponents.

$$\left(\frac{x}{4}\right)^{-4} = \frac{x^{-4}}{4^{-4}} \quad \text{Power of a Quotient Property}$$

$$= \frac{4^4}{x^4} \quad \text{Definition of negative exponent}$$

$$= \frac{256}{x^4} \quad \text{Simplify.}$$

**Simplify the expression. Write your answer using only positive exponents.**

**1.** $y^3 \cdot y^{-5}$ **2.** $\frac{x^4}{x^7}$ **3.** $(x^0y^2)^3$ **4.** $\left(\frac{2x^2}{5y^4}\right)^{-2}$

## 6.2 Radicals and Rational Exponents

(pp. 299–304)

Evaluate $512^{1/3}$.

$$512^{1/3} = \sqrt[3]{512} \quad \text{Rewrite the expression in radical form.}$$

$$= \sqrt[3]{8 \cdot 8 \cdot 8} \quad \text{Rewrite the expression showing factors.}$$

$$= 8 \quad \text{Evaluate the cube root.}$$

**Evaluate the expression.**

**5.** $\sqrt[3]{8}$ **6.** $\sqrt[5]{-243}$ **7.** $625^{3/4}$ **8.** $(-25)^{1/2}$

## 6.3 Exponential Functions

(pp. 305–312)

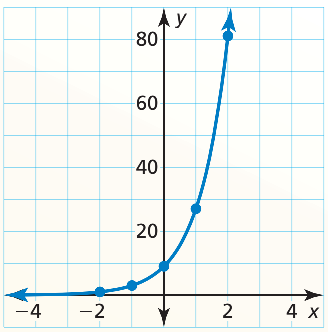
Graph $f(x) = 9(3)^x$.

**Step 1** Make a table of values.

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **f(x)** | 1 | 3 | 9 | 27 | 81 |

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.




**Graph the function. Describe the domain and range.**

**9.** $f(x) = -4\left(\frac{1}{4}\right)^x$ **10.** $f(x) = 3^{x+2}$ **11.** $f(x) = 2^{x-4} - 3$

**12.** Write and graph an exponential function $f$ represented by the table. Then compare the graph to the graph of $g(x) = \left(\frac{1}{2}\right)^x$.

| **x** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **y** | 2 | 1 | 0.5 | 0.25 |

## 6.4 Exponential Growth and Decay (pp. 313–322)

Rewrite the function $y = 10(0.65)^{t/8}$ to determine whether it represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

$y = 10(0.65)^{t/8}$ — Write the function.

$= 10(0.65^{1/8})^t$ — Power of a Power Property

$\approx 10(0.95)^t$ — Evaluate the power.

The function is of the form $y = a(1 - r)^t$, where $1 - r < 1$, so it represents exponential decay. Use the decay factor $1 - r$ to find the rate of decay.

$1 - r = 0.95$ — Write an equation.

$r = 0.05$ — Solve for $r$.

So, the function represents exponential decay, and the rate of decay is 5%.

**Determine whether the table represents an *exponential growth function,* an *exponential decay function,* or *neither.* Explain.**

**13.**

| *x* | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| *y* | 3 | 6 | 12 | 24 |

**14.**

| *x* | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *y* | 162 | 108 | 72 | 48 |

**Rewrite the function to determine whether it represents *exponential growth* or *exponential decay.* Identify the percent rate of change.**

**15.** $f(t) = 4(1.25)^{t+3}$

**16.** $y = (1.06)^{8t}$

**17.** $f(t) = 6(0.84)^{t-4}$

**18.** You deposit \$750 in a savings account that earns 5% annual interest compounded quarterly. (a) Write a function that represents the balance after $t$ years. (b) What is the balance of the account after 4 years?

**19.** The value of a TV is \$1500. Its value decreases by 14% each year. (a) Write a function that represents the value $y$ (in dollars) of the TV after $t$ years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the TV after 3 years.

## 6.5 Solving Exponential Equations (pp. 325–330)

Solve $\frac{1}{9} = 3^{x+6}$.

$\frac{1}{9} = 3^{x+6}$ — Write the equation.

$3^{-2} = 3^{x+6}$ — Rewrite $\frac{1}{9}$ as $3^{-2}$.

$-2 = x + 6$ — Equate the exponents.

$x = -8$ — Solve for $x$.

**Solve the equation.**

**20.** $5^x = 5^{3x-2}$

**21.** $3^{x-2} = 1$

**22.** $-4 = 6^{4x-3}$

**23.** $\left(\frac{1}{3}\right)^{2x+3} = 5$

**24.** $\left(\frac{1}{16}\right)^{3x} = 64^{2(x+8)}$

**25.** $27^{2x+2} = 81^{x+4}$

## 6.6 Geometric Sequences (pp. 331–338)

Write the next three terms of the geometric sequence 2, 6, 18, 54, . . ..

Use a table to organize the terms and extend the sequence.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Term | 2 | 6 | 18 | 54 | 162 | 486 | 1458 |

$\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$

Each term is 3 times the previous term. So, the common ratio is 3.

Multiply a term by 3 to find the next term.

▶ The next three terms are 162, 486, and 1458.

**Decide whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. If the sequence is geometric, write the next three terms and graph the sequence.**

**26.** 3, 12, 48, 192, . . .

**27.** 9, −18, 27, −36, . . .

**28.** 375, −75, 15, −3, . . .

**Write an equation for the *n*th term of the geometric sequence. Then find $a_9$.**

**29.** 1, 4, 16, 64, . . .

**30.** 5, −10, 20, −40, . . .

**31.** 486, 162, 54, 18, . . .

## 6.7 Recursively Defined Sequences (pp. 339–346)

Write a recursive rule for the sequence 5, 12, 19, 26, 33, . . ..

Use a table to organize the terms and find the pattern.

| Position, $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Term, $a_n$ | 5 | 12 | 19 | 26 | 33 |

$+7$ $+7$ $+7$ $+7$

The sequence is arithmetic, with first term $a_1 = 5$ and common difference $d = 7$.

$a_n = a_{n-1} + d$ Recursive equation for an arithmetic sequence

$a_n = a_{n-1} + 7$ Substitute 7 for *d*.

▶ So, a recursive rule for the sequence is $a_1 = 5$, $a_n = a_{n-1} + 7$.

**Write the first six terms of the sequence. Then graph the sequence.**

**32.** $a_1 = 4$, $a_n = a_{n-1} + 5$

**33.** $a_1 = -4$, $a_n = -3a_{n-1}$

**34.** $a_1 = 32$, $a_n = \frac{1}{4}a_{n-1}$

**Write a recursive rule for the sequence.**

**35.** 3, 8, 13, 18, 23, . . .

**36.** 3, 6, 12, 24, 48, . . .

**37.** 7, 6, 13, 19, 32, . . .

**38.** The first term of a sequence is 8. Each term of the sequence is 5 times the preceding term. Graph the first four terms of the sequence. Write a recursive rule and an explicit rule for the sequence.

# 6 Chapter Test

**Evaluate the expression.**

**1.** $-\sqrt[4]{16}$

**2.** $729^{1/6}$

**3.** $(-32)^{7/5}$

**Simplify the expression. Write your answer using only positive exponents.**

**4.** $z^{-2} \cdot z^{4}$

**5.** $\dfrac{b^{-5}}{a^{0}b^{-8}}$

**6.** $\left(\dfrac{2c^{4}}{5}\right)^{-3}$

**Write and graph a function that represents the situation.**

**7.** Your starting annual salary of $42,500 increases by 3% each year.

**8.** You deposit $500 in an account that earns 6.5% annual interest compounded yearly.

**Write an explicit rule and a recursive rule for the sequence.**

**9.**

| $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $a_n$ | −6 | 8 | 22 | 36 |

**10.**

| $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $a_n$ | 400 | 100 | 25 | 6.25 |

**Solve the equation. Check your solution.**

**11.** $2^{x} = \dfrac{1}{128}$

**12.** $256^{x+2} = 16^{3x-1}$

**13.** Graph $f(x) = 2(6)^{x}$. Compare the graph to the graph of $g(x) = 6^{x}$. Describe the domain and range of $f$.

**Use the equation to complete the statement "$a$ □ $b$" with the symbol $<$, $>$, or $=$. Do not attempt to solve the equation.**

**14.** $\dfrac{5^{a}}{5^{b}} = 5^{-3}$

**15.** $9^{a} \cdot 9^{-b} = 1$

**16.** The first two terms of a sequence are $a_1 = 3$ and $a_2 = -12$. Let $a_3$ be the third term when the sequence is arithmetic and let $b_3$ be the third term when the sequence is geometric. Find $a_3 - b_3$.

**17.** At sea level, Earth's atmosphere exerts a pressure of 1 atmosphere. Atmospheric pressure $P$ (in atmospheres) decreases with altitude. It can be modeled by $P = (0.99988)^{a}$, where $a$ is the altitude (in meters).

**a.** Identify the initial amount, decay factor, and decay rate.

**b.** Use a graphing calculator to graph the function. Use the graph to estimate the atmospheric pressure at an altitude of 5000 feet.

**18.** You follow the training schedule from your coach.

**a.** Write an explicit rule and a recursive rule for the geometric sequence.

**b.** On what day do you run approximately 3 kilometers?

**Training On Your Own**

**Day 1:** Run 1 km.

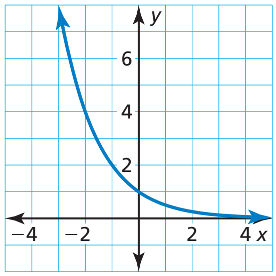
**Each day after Day 1:** Run 20% farther than the previous day.

# 6 Cumulative Assessment

**1.** Fill in the exponent of *x* with a number to simplify the expression.

$$\frac{x^{5/3} \cdot x^{-1} \cdot \sqrt[3]{x}}{x^{-2} \cdot x^{0}} = x^{\square}$$

**2.** The graph of the exponential function *f* is shown. Find $f(-7)$.




**3.** Student A claims he can form a linear system from the equations shown that has infinitely many solutions. Student B claims she can form a linear system from the equations shown that has one solution. Student C claims he can form a linear system from the equations shown that has no solution.

| | | |
|---|---|---|
| $3x + y = 12$ | $3x + 2y = 12$ | $6x + 2y = 6$ |
| $3y + 9x = 36$ | $2y - 6x = 12$ | $9x - 3y = -18$ |

**a.** Select two equations to support Student A's claim.

**b.** Select two equations to support Student B's claim.

**c.** Select two equations to support Student C's claim.

**4.** Fill in the inequality with <, ≤, >, or ≥ so that the system of linear inequalities has no solution.

**Inequality 1** $y - 2x \le 4$

**Inequality 2** $6x - 3y \;\square\; -12$

**5.** The second term of a sequence is 7. Each term of the sequence is 10 more than the preceding term. Fill in values to write a recursive rule and an explicit rule for the sequence.

$a_1 = \square$, $a_n = a_{n-1} + \square$

$a_n = \square n - \square$

**6.** A data set consists of the heights $y$ (in feet) of a hot-air balloon $t$ minutes after it begins its descent. An equation of the line of best fit is $y = 870 - 14.8t$. Which of the following is a correct interpretation of the line of best fit?

Ⓐ The initial height of the hot-air balloon is 870 feet. The slope has no meaning in this context.

Ⓑ The initial height of the hot-air balloon is 870 feet, and it descends 14.8 feet per minute.

Ⓒ The initial height of the hot-air balloon is 870 feet, and it ascends 14.8 feet per minute.

Ⓓ The hot-air balloon descends 14.8 feet per minute. The $y$-intercept has no meaning in this context.

**7.** Select all the functions whose $x$-value is an integer when $f(x) = 10$.

| | | |
|---|---|---|
| $f(x) = 3x - 2$ | $f(x) = -2x + 4$ | $f(x) = \frac{3}{2}x + 4$ |
| $f(x) = -3x + 5$ | $f(x) = \frac{1}{2}x - 6$ | $f(x) = 4x + 14$ |

**8.** Place each function into one of the three categories. For exponential functions, state whether the function represents *exponential growth*, *exponential decay*, or *neither.*

| Exponential | Linear | Neither |
|---|---|---|
| | | |

| | | |
|---|---|---|
| $f(x) = -2(8)^x$ | $f(x) = 15 - x$ | $f(x) = \frac{1}{2}(3)^x$ |
| $f(x) = 6x^2 + 9$ | $f(x) = 4(1.6)^{x/10}$ | $f(x) = x(18 - x)$ |
| $f(x) = 3\left(\frac{1}{6}\right)^x$ | $f(x) = -3(4x + 1 - x)$ | $f(x) = \sqrt[4]{16} + 2x$ |

**9.** How does the graph shown compare to the graph of $f(x) = 2^x$?