

2.5 Solving Compound Inequalities

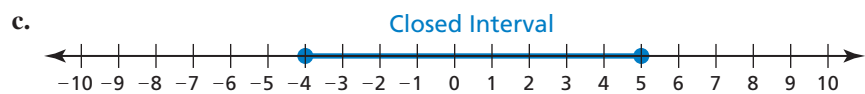
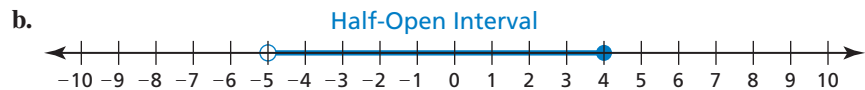
Essential Question How can you use inequalities to describe intervals on the real number line?

EXPLORATION 1 Describing Intervals on the Real Number Line

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

REASONING ABSTRACTLY

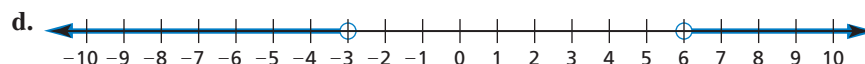
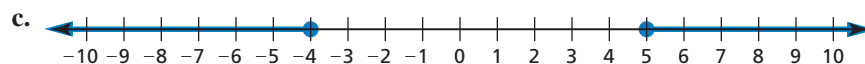
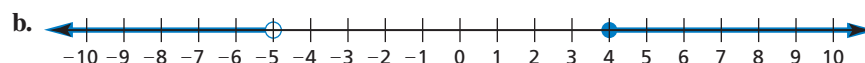
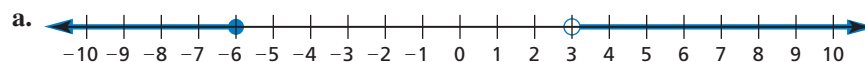
To be proficient in math, you need to create a clear representation of the problem at hand.



e. Do you use “and” or “or” to connect the two inequalities in parts (a)–(d)? Explain.

EXPLORATION 2 Describing Two Infinite Intervals

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.



e. Do you use “and” or “or” to connect the two inequalities in parts (a)–(d)? Explain.

Communicate Your Answer

3. How can you use inequalities to describe intervals on the real number line?

2.5 Lesson

Core Vocabulary

compound inequality, p. 82

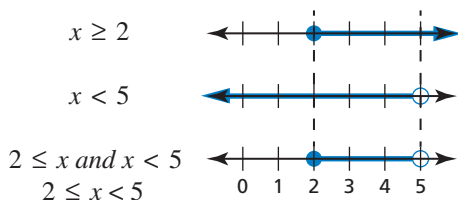
What You Will Learn

- ▶ Write and graph compound inequalities.
- ▶ Solve compound inequalities.
- ▶ Use compound inequalities to solve real-life problems.

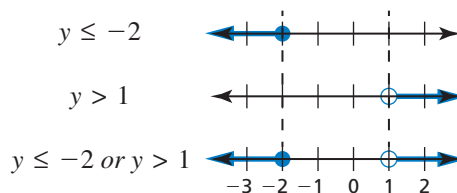
Writing and Graphing Compound Inequalities

A **compound inequality** is an inequality formed by joining two inequalities with the word “and” or the word “or.”

The graph of a compound inequality with “and” is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.



The graph of a compound inequality with “or” is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.



EXAMPLE 1 Writing and Graphing Compound Inequalities

Write each sentence as an inequality. Graph each inequality.

- a. A number x is greater than -8 and less than or equal to 4 .
- b. A number y is at most 0 or at least 2 .

SOLUTION

- a. A **number x is greater than -8** and **less than or equal to 4** .

$$x > -8 \quad \text{and} \quad x \leq 4$$

- ▶ An inequality is $-8 < x \leq 4$.

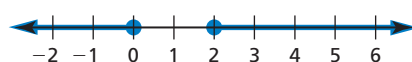


Graph the intersection of the graphs of $x > -8$ and $x \leq 4$.

- b. A **number y is at most 0** or **at least 2** .

$$y \leq 0 \quad \text{or} \quad y \geq 2$$

- ▶ An inequality is $y \leq 0$ or $y \geq 2$.



Graph the union of the graphs of $y \leq 0$ and $y \geq 2$.

REMEMBER

A compound inequality with “and” can be written as a single inequality. For example, you can write $x > -8$ and $x \leq 4$ as $-8 < x \leq 4$.

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Write the sentence as an inequality. Graph the inequality.

- A number d is more than 0 and less than 10 .
- A number a is fewer than -6 or no less than -3 .

LOOKING FOR STRUCTURE

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with “and” is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

EXAMPLE 2 Solving Compound Inequalities with “And”

Solve each inequality. Graph each solution.

a. $-4 < x - 2 < 3$

b. $-3 < -2x + 1 \leq 9$

SOLUTION

a. Separate the compound inequality into two inequalities, then solve.

$$-4 < x - 2 \quad \text{and} \quad x - 2 < 3 \quad \text{Write two inequalities.}$$

$$\begin{array}{r} +2 \\ -2 < x \end{array} \quad \text{and} \quad \begin{array}{r} +2 \\ x < 5 \end{array} \quad \text{Add 2 to each side.}$$

$$-2 < x \quad \text{and} \quad x < 5 \quad \text{Simplify.}$$

▶ The solution is $-2 < x < 5$. 

b. $-3 < -2x + 1 \leq 9$

Write the inequality.

$$\begin{array}{r} -1 \\ -4 < -2x \end{array} \leq \begin{array}{r} -1 \\ 8 \end{array} \quad \text{Subtract 1 from each expression.}$$

$$-4 < -2x \leq 8 \quad \text{Simplify.}$$

$$\begin{array}{r} -4 \\ -2 > -2x \end{array} \geq \begin{array}{r} 8 \\ -2 \end{array} \quad \text{Divide each expression by } -2. \\ \text{Reverse each inequality symbol.}$$

$$2 > x \geq -4 \quad \text{Simplify.}$$

▶ The solution is $-4 \leq x < 2$. 

EXAMPLE 3 Solving a Compound Inequality with “Or”

Solve $3y - 5 < -8$ or $2y - 1 > 5$. Graph the solution.

SOLUTION

$$3y - 5 < -8 \quad \text{or} \quad 2y - 1 > 5 \quad \text{Write the inequality.}$$

$$\begin{array}{r} +5 \\ 3y < -3 \end{array} \quad \text{or} \quad \begin{array}{r} +1 \\ 2y > 6 \end{array} \quad \text{Addition Property of Inequality}$$

$$3y < -3 \quad \text{or} \quad 2y > 6 \quad \text{Simplify.}$$

$$\begin{array}{r} 3y \\ 3 < -3 \end{array} \quad \text{or} \quad \begin{array}{r} 2y \\ 2 > 6 \end{array} \quad \text{Division Property of Inequality}$$

$$y < -1 \quad \text{or} \quad y > 3 \quad \text{Simplify.}$$

▶ The solution is $y < -1$ or $y > 3$. 

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Solve the inequality. Graph the solution.

3. $5 \leq m + 4 < 10$

4. $-3 < 2k - 5 < 7$

5. $4c + 3 \leq -5$ or $c - 8 > -1$

6. $2p + 1 < -7$ or $3 - 2p \leq -1$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics



Operating temperature:
0°C to 35°C

Electrical devices should operate effectively within a specified temperature range. Outside the operating temperature range, the device may fail.

- Write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone.
- Describe one situation in which the surrounding temperature could be below the operating range and one in which it could be above.

SOLUTION

- Understand the Problem** You know the operating temperature range in degrees Celsius. You are asked to write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then you are asked to describe situations outside this range.
- Make a Plan** Write a compound inequality in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit. Then solve the inequality and describe the situations.
- Solve the Problem** Let C be the temperature in degrees Celsius, and let F be the temperature in degrees Fahrenheit.

$$0 \leq C \leq 35$$

Write the inequality using C .

$$0 \leq \frac{5}{9}(F - 32) \leq 35$$

Substitute $\frac{5}{9}(F - 32)$ for C .

$$9 \cdot 0 \leq 9 \cdot \frac{5}{9}(F - 32) \leq 9 \cdot 35$$

Multiply each expression by 9.

$$0 \leq 5(F - 32) \leq 315$$

Simplify.

$$0 \leq 5F - 160 \leq 315$$

Distributive Property

$$\begin{array}{r} + 160 \\ 160 \leq 5F - 160 \leq 315 \\ + 160 \end{array}$$

Add 160 to each expression.

$$160 \leq 5F \leq 475$$

Simplify.

$$\frac{160}{5} \leq \frac{5F}{5} \leq \frac{475}{5}$$

Divide each expression by 5.

$$32 \leq F \leq 95$$

Simplify.

- The solution is $32 \leq F \leq 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. One situation when the surrounding temperature could be below this range is winter in Alaska. One situation when the surrounding temperature could be above this range is daytime in the Mojave Desert of the American Southwest.

- Look Back** You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for F in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius.

STUDY TIP

You can also solve the inequality by first multiplying each expression by $\frac{9}{5}$.



−40°C to 15°C

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- Write and solve a compound inequality that represents the temperature rating (in degrees Fahrenheit) of the winter boots.

Vocabulary and Core Concept Check

- WRITING** Compare the graph of $-6 \leq x \leq -4$ with the graph of $x \leq -6$ or $x \geq -4$.
- WHICH ONE DOESN'T BELONG?** Which compound inequality does *not* belong with the other three? Explain your reasoning.

$$a > 4 \text{ or } a < -3$$

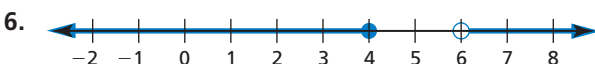
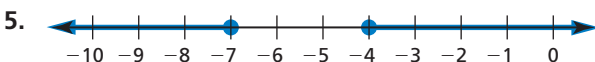
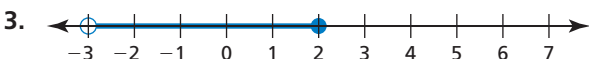
$$a < -2 \text{ or } a > 8$$

$$a > 7 \text{ or } a < -5$$

$$a < 6 \text{ or } a > -9$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write a compound inequality that is represented by the graph.

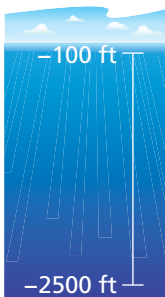


In Exercises 7–10, write the sentence as an inequality. Graph the inequality. (See Example 1.)

- A number p is less than 6 and greater than 2.
- A number n is less than or equal to -7 or greater than 12.
- A number m is more than $-7\frac{2}{3}$ or at most -10 .
- A number r is no less than -1.5 and fewer than 9.5.

11. **MODELING WITH MATHEMATICS**

Slitsnails are large mollusks that live in deep waters. They have been found in the range of elevations shown. Write and graph a compound inequality that represents this range.



12. **MODELING WITH MATHEMATICS** The life zones on Mount Rainier, a mountain in Washington, can be approximately classified by elevation, as follows.

Low-elevation forest: above 1700 feet to 2500 feet

Mid-elevation forest: above 2500 feet to 4000 feet

Subalpine: above 4000 feet to 6500 feet

Alpine: above 6500 feet to the summit



Elevation of Mount Rainier: 14,410 ft


Write a compound inequality that represents the elevation range for each type of plant life.

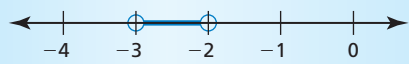
- trees in the low-elevation forest zone
- flowers in the subalpine and alpine zones


In Exercises 13–20, solve the inequality. Graph the solution. (See Examples 2 and 3.)

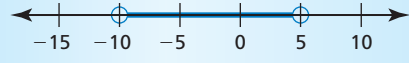
- $6 < x + 5 \leq 11$
- $24 > -3r \geq -9$
- $v + 8 < 3$ or $-8v < -40$
- $-14 > w + 3$ or $3w \geq -27$
- $2r + 3 < 7$ or $-r + 9 \leq 2$
- $-6 < 3n + 9 < 21$
- $-12 < \frac{1}{2}(4x + 16) < 18$
- $35 < 7(2 - b)$ or $\frac{1}{3}(15b - 12) \geq 21$

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the inequality or graphing the solution.

21.  $4 < -2x + 3 < 9$
 $4 < -2x < 6$
 $-2 > x > -3$



22.  $x - 2 > 3$ or $x + 8 < -2$
 $x > 5$ or $x < -10$



23. MODELING WITH MATHEMATICS


Write and solve a compound inequality that represents the possible temperatures (in degrees Fahrenheit) of the interior of the iceberg. (See Example 4.)



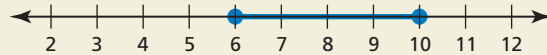
24. PROBLEM SOLVING A ski shop sells skis with lengths ranging from 150 centimeters to 220 centimeters. The shop says the length of the skis should be about 1.16 times a skier's height (in centimeters). Write and solve a compound inequality that represents the heights of skiers the shop does *not* provide skis for.

In Exercises 25–30, solve the inequality. Graph the solution, if possible.

- 25. $22 < -3c + 4 < 14$
- 26. $2m - 1 \geq 5$ or $5m > -25$
- 27. $-y + 3 \leq 8$ and $y + 2 > 9$
- 28. $x - 8 \leq 4$ or $2x + 3 > 9$
- 29. $2n + 19 \leq 10 + n$ or $-3n + 3 < -2n + 33$

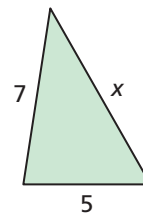
- 30. $3x - 18 < 4x - 23$ and $x - 16 < -22$
- 31. **REASONING** Fill in the compound inequality $4(x - 6)$  $2(x - 10)$ and $5(x + 2) \geq 2(x + 8)$ with $<$, \leq , $>$, or \geq so that the solution is only one value.

32. THOUGHT PROVOKING Write a real-life story that can be modeled by the graph.

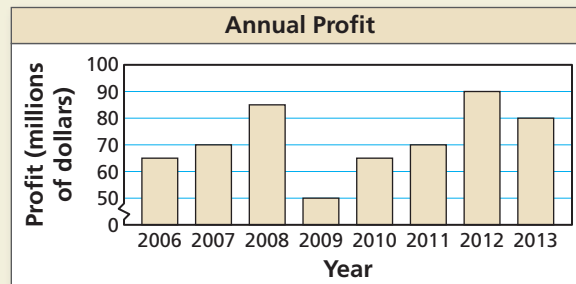


33. MAKING AN ARGUMENT

The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Use the triangle shown to write and solve three inequalities. Your friend claims the value of x can be 1. Is your friend correct? Explain.



34. HOW DO YOU SEE IT? The graph shows the annual profits of a company from 2006 to 2013.



- a. Write a compound inequality that represents the annual profits from 2006 to 2013.
- b. You can use the formula $P = R - C$ to find the profit P , where R is the revenue and C is the cost. From 2006 to 2013, the company's annual cost was about \$125 million. Is it possible the company had an annual revenue of \$160 million from 2006 to 2013? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Graph the solutions, if possible. (Section 1.4)

- 35. $\left| \frac{d}{9} \right| = 6$
- 36. $7|5p - 7| = -21$
- 37. $|r + 2| = |3r - 4|$
- 38. $\left| \frac{1}{2}w - 6 \right| = |w + 7|$

Find and interpret the mean absolute deviation of the data. (Skills Review Handbook)

- 39. 1, 1, 2, 5, 6, 8, 10, 12, 12, 13
- 40. 24, 26, 28, 28, 30, 30, 32, 32, 34, 36