3.4 Graphing Linear Equations in Standard Form

Essential Question

How can you describe the graph of the equation $Ax + By = C$?

EXPLORATION 1 Using a Table to Plot Points

Work with a partner. You sold a total of \$16 worth of tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are \$4 each. Child tickets are \$2 each.

$$\frac{\square}{\text{adult}} \cdot \text{Number of adult tickets} + \frac{\square}{\text{child}} \cdot \text{Number of child tickets} = \square$$

a. Let x represent the number of adult tickets. Let y represent the number of child tickets. Use the verbal model to write an equation that relates x and y.

b. Copy and complete the table to show the different combinations of tickets you might have sold.

x					
y					

c. Plot the points from the table. Describe the pattern formed by the points.

d. If you remember how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

FINDING AN ENTRY POINT

To be proficient in math, you need to find an entry point into the solution of a problem. Determining what information you know, and what you can do with that information, can help you find an entry point.

EXPLORATION 2 Rewriting and Graphing an Equation

Work with a partner. You sold a total of \$48 worth of cheese. You forgot how many pounds of each type of cheese you sold. Swiss cheese costs \$8 per pound. Cheddar cheese costs \$6 per pound.

$$\frac{\square}{\text{pound}} \cdot \text{Pounds of Swiss} + \frac{\square}{\text{pound}} \cdot \text{Pounds of cheddar} = \square$$

a. Let x represent the number of pounds of Swiss cheese. Let y represent the number of pounds of cheddar cheese. Use the verbal model to write an equation that relates x and y.

b. Solve the equation for y. Then use a graphing calculator to graph the equation. Given the real-life context of the problem, find the domain and range of the function.

c. The **x-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. The **y-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. Use the graph to determine the x- and y-intercepts.

d. How could you use the equation you found in part (a) to determine the x- and y-intercepts? Explain your reasoning.

e. Explain the meaning of the intercepts in the context of the problem.

Communicate Your Answer

3. How can you describe the graph of the equation $Ax + By = C$?
4. Write a real-life problem that is similar to those shown in Explorations 1 and 2.

3.4 Lesson

Core Vocabulary

standard form, *p. 130*
x-intercept, *p. 131*
y-intercept, *p. 131*

Previous
ordered pair
quadrant

What You Will Learn

- Graph equations of horizontal and vertical lines.
- Graph linear equations in standard form using intercepts.
- Use linear equations in standard form to solve real-life problems.

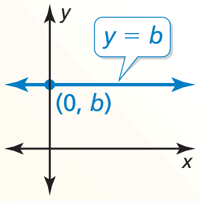
Horizontal and Vertical Lines

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

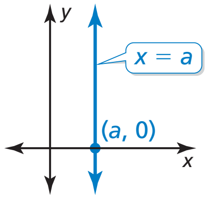
Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

Core Concept

Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



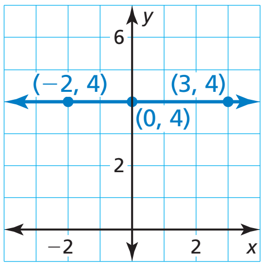
The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

EXAMPLE 1 Horizontal and Vertical Lines

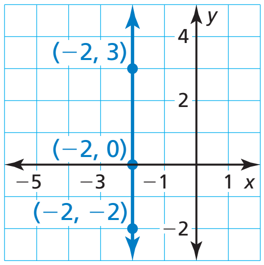
Graph (a) $y = 4$ and (b) $x = -2$.

SOLUTION

a. For every value of x, the value of y is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the x-axis.



b. For every value of y, the value of x is -2. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the y-axis.



STUDY TIP

For every value of x, the ordered pair $(x, 4)$ is a solution of $y = 4$.

Monitoring Progress

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Graph the linear equation.

1. $y = -2.5$

2. $x = 5$

Using Intercepts to Graph Linear Equations

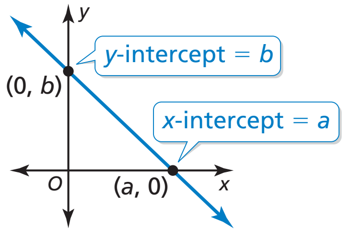
You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the points where the graph crosses the axes.

Core Concept

Using Intercepts to Graph Equations

The ***x*-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. It occurs when $y = 0$.

The ***y*-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. It occurs when $x = 0$.



To graph the linear equation $Ax + By = C$, find the intercepts and draw the line that passes through the two intercepts.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.

EXAMPLE 2 Using Intercepts to Graph a Linear Equation

Use intercepts to graph the equation $3x + 4y = 12$.

SOLUTION

Step 1 Find the intercepts.

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$ — Write the original equation.

$3x + 4(0) = 12$ — Substitute 0 for y.

$x = 4$ — Solve for x.

To find the y-intercept, substitute 0 for x and solve for y.

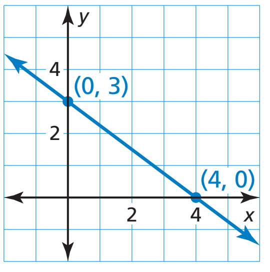
$3x + 4y = 12$ — Write the original equation.

$3(0) + 4y = 12$ — Substitute 0 for x.

$y = 3$ — Solve for y.

Step 2 Plot the points and draw the line.

The x-intercept is 4, so plot the point (4, 0). The y-intercept is 3, so plot the point (0, 3). Draw a line through the points.



STUDY TIP

As a check, you can find a third solution of the equation and verify that the corresponding point is on the graph. To find a third solution, substitute any value for one of the variables and solve for the other variable.

Monitoring Progress

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Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

3. $2x - y = 4$

4. $x + 3y = -9$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

You are planning an awards banquet for your school. You need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where x is the number of small tables and y is the number of large tables.

a. Graph the equation. Interpret the intercepts.

b. Find four possible solutions in the context of the problem.

SOLUTION

1. **Understand the Problem** You know the equation that models the situation. You are asked to graph the equation, interpret the intercepts, and find four solutions.

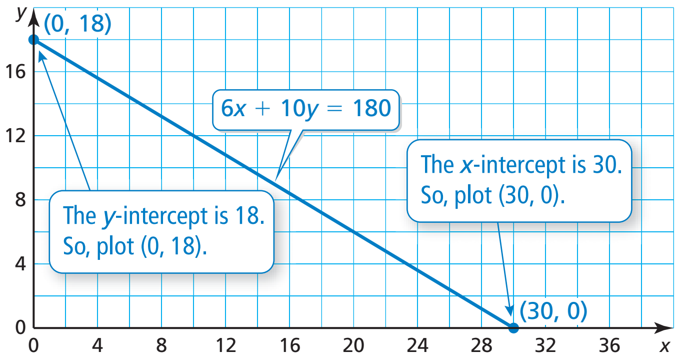
2. **Make a Plan** Use intercepts to graph the equation. Then use the graph to interpret the intercepts and find other solutions.

3. **Solve the Problem**

a. Use intercepts to graph the equation. Neither x nor y can be negative, so only graph the equation in the first quadrant.

STUDY TIP

Although x and y represent whole numbers, it is convenient to draw a line segment that includes points whose coordinates are not whole numbers.



▶ The x-intercept shows that you can rent 30 small tables when you do not rent any large tables. The y-intercept shows that you can rent 18 large tables when you do not rent any small tables.

b. Only whole-number values of x and y make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the points (10, 12) and (20, 6). To verify that these points are solutions, check them in the equation, as shown.

Check

$$6x + 10y = 180$$

$$6(10) + 10(12) \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

$$6x + 10y = 180$$

$$6(20) + 10(6) \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

▶ So, four possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, 20 small and 6 large, and 30 small and 0 large.

4. **Look Back** The graph shows that as the number x of small tables increases, the number y of large tables decreases. This makes sense in the context of the problem. So, the graph is reasonable.

Monitoring Progress

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5. WHAT IF? You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where x is the number of small tables and y is the number of large tables. Graph the equation and interpret the intercepts.

3.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are x-intercepts and y-intercepts alike? How are they different?

2. **WHICH ONE DOESN'T BELONG?** Which point does not belong with the other three? Explain your reasoning.

$(0, -3)$	$(0, 0)$	$(4, -3)$	$(4, 0)$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the linear equation. *(See Example 1.)*

3. $x = 4$
4. $y = 2$
5. $y = -3$
6. $x = -1$

In Exercises 7–12, find the x- and y-intercepts of the graph of the linear equation.

7. $2x + 3y = 12$
8. $3x + 6y = 24$
9. $-4x + 8y = -16$
10. $-6x + 9y = -18$
11. $3x - 6y = 2$
12. $-x + 8y = 4$

In Exercises 13–22, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. *(See Example 2.)*

13. $5x + 3y = 30$
14. $4x + 6y = 12$
15. $-12x + 3y = 24$
16. $-2x + 6y = 18$
17. $-4x + 3y = -30$
18. $-2x + 7y = -21$
19. $-x + 2y = 7$
20. $3x - y = -5$
21. $-\frac{5}{2}x + y = 10$
22. $-\frac{1}{2}x + y = -4$

23. **MODELING WITH MATHEMATICS** A football team has an away game, and the bus breaks down. The coaches decide to drive the players to the game in cars and vans. Four players can ride in each car. Six players can ride in each van. There are 48 players on the team. The equation $4x + 6y = 48$ models this situation, where x is the number of cars and y is the number of vans. *(See Example 3.)*

    a. Graph the equation. Interpret the intercepts.

    b. Find four possible solutions in the context of the problem.

24. **MODELING WITH MATHEMATICS** You are ordering shirts for the math club at your school. Short-sleeved shirts cost \$10 each. Long-sleeved shirts cost \$12 each. You have a budget of \$300 for the shirts. The equation $10x + 12y = 300$ models the total cost, where x is the number of short-sleeved shirts and y is the number of long-sleeved shirts.

    a. Graph the equation. Interpret the intercepts.

    b. Twelve students decide they want short-sleeved shirts. How many long-sleeved shirts can you order?

ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in finding the intercepts of the graph of the equation.

25.

$3x + 12y = 24$	$3x + 12y = 24$
$3x + 12(0) = 24$	$3(0) + 12y = 24$
$3x = 24$	$12y = 24$
$x = 8$	$y = 2$

The intercept is at $(8, 2)$.

26.

$4x + 10y = 20$	$4x + 10y = 20$
$4x + 10(0) = 20$	$4(0) + 10y = 20$
$4x = 20$	$10y = 20$
$x = 5$	$y = 2$

The x-intercept is at $(0, 5)$, and the y-intercept is at $(2, 0)$.

27. MAKING AN ARGUMENT You overhear your friend explaining how to find intercepts to a classmate. Your friend says, "When you want to find the x-intercept, just substitute 0 for x and continue to solve the equation." Is your friend's explanation correct? Explain.

28. ANALYZING RELATIONSHIPS You lose track of how many 2-point baskets and 3-point baskets a team makes in a basketball game. The team misses all the 1-point baskets and still scores 54 points. The equation $2x + 3y = 54$ models the total points scored, where x is the number of 2-point baskets made and y is the number of 3-point baskets made.

a. Find and interpret the intercepts.

b. Can the number of 3-point baskets made be odd? Explain your reasoning.

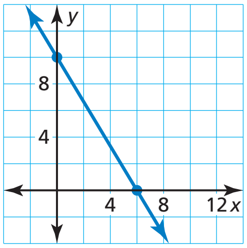
c. Graph the equation. Find two more possible solutions in the context of the problem.

MULTIPLE REPRESENTATIONS **In Exercises 29–32, match the equation with its graph.**

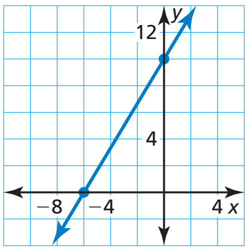
29. $5x + 3y = 30$ **30.** $5x + 3y = -30$

31. $5x - 3y = 30$ **32.** $5x - 3y = -30$

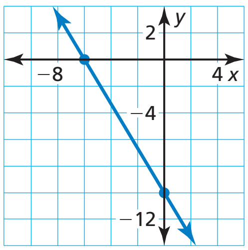
A.



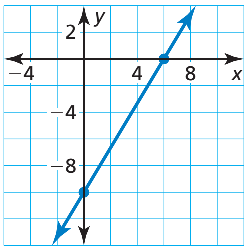
B.



C.

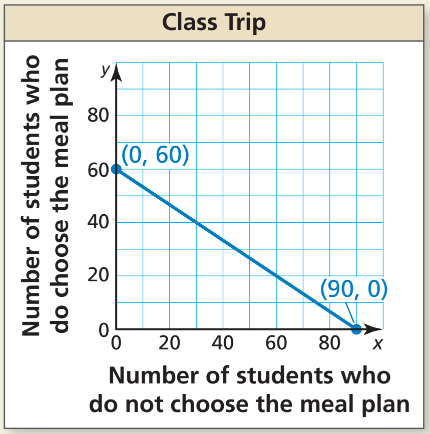


D.



33. MATHEMATICAL CONNECTIONS Graph the equations $x = 5$, $x = 2$, $y = -2$, and $y = 1$. What enclosed shape do the lines form? Explain your reasoning.

34. HOW DO YOU SEE IT? You are organizing a class trip to an amusement park. The cost to enter the park is \$30. The cost to enter with a meal plan is \$45. You have a budget of \$2700 for the trip. The equation $30x + 45y = 2700$ models the total cost for the class to go on the trip, where x is the number of students who do not choose the meal plan and y is the number of students who do choose the meal plan.



a. Interpret the intercepts of the graph.

b. Describe the domain and range in the context of the problem.

35. REASONING Use the values to fill in the equation $\square x + \square y = 30$ so that the x-intercept of the graph is -10 and the y-intercept of the graph is 5.

-10 -3 1 5 6

36. THOUGHT PROVOKING Write an equation in standard form of a line whose intercepts are integers. Explain how you know the intercepts are integers.

37. WRITING Are the equations of horizontal and vertical lines written in standard form? Explain your reasoning.

38. ABSTRACT REASONING The x- and y-intercepts of the graph of the equation $3x + 5y = k$ are integers. Describe the values of k. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

39. $\dfrac{2-(-2)}{4-(-4)}$ **40.** $\dfrac{14-18}{0-2}$ **41.** $\dfrac{-3-9}{8-(-7)}$ **42.** $\dfrac{12-17}{-5-(-2)}$