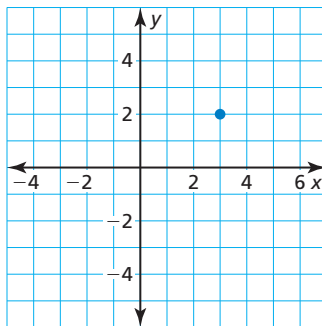# Writing Equations in Point-Slope Form

Essential Question How can you write an equation of a line when you are given the slope and a point on the line?
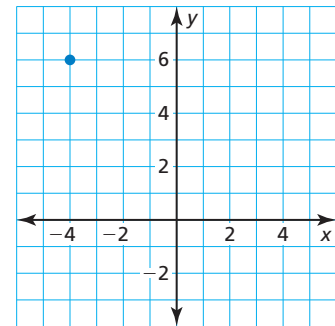
EXPLORATION 1 Writing Equations of Lines

Work with a partner.
- Sketch the line that has the given slope and passes through the given point.
- Find the *y*-intercept of the line.
- Write an equation of the line.

a. $m = \frac{1}{2}$

b. $m = -2$

EXPLORATION 2 Writing a Formula

Work with a partner.

The point (x_1, y_1) is a given point on a nonvertical line. The point (x, y) is any other point on the line. Write an equation that represents the slope m of the line. Then rewrite this equation by multiplying each side by the difference of the *x*-coordinates to obtain the **point-slope form** of a linear equation.
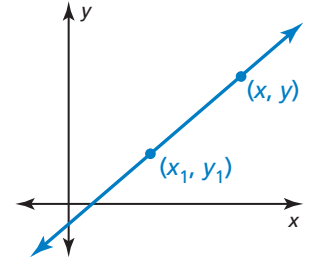
USING A GRAPHING CALCULATOR

To be proficient in math, you need to understand the feasibility, appropriateness, and limitations of the technological tools at your disposal. For instance, in real-life situations such as the one given in Exploration 3, it may not be feasible to use a square viewing window on a graphing calculator.

EXPLORATION 3 Writing an Equation

Work with a partner.

For four months, you have saved $25 per month. You now have $175 in your savings account.

a. Use your result from Exploration 2 to write an equation that represents the balance *A* after *t* months.

b. Use a graphing calculator to verify your equation.
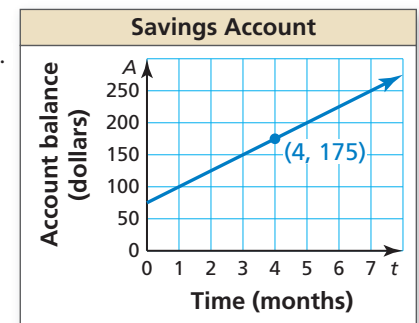
Communicate Your Answer

4. How can you write an equation of a line when you are given the slope and a point on the line?

5. Give an example of how to write an equation of a line when you are given the slope and a point on the line. Your example should be different from those above.

What You Will Learn

▶ Write an equation of a line given its slope and a point on the line.

▶ Write an equation of a line given two points on the line.

▶ Use linear equations to solve real-life problems.

Core Vocabulary

point-slope form, *p. 182*

Previous
slope-intercept form
function
linear model
rate

Writing Equations of Lines in Point-Slope Form

Given a point on a line and the slope of the line, you can write an equation of the line. Consider the line that passes through $(2, 3)$ and has a slope of $\frac{1}{2}$. Let (x, y) be another point on the line where $x \neq 2$. You can write an equation relating x and y using the slope formula with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, y)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Write the slope formula.

$$\frac{1}{2} = \frac{y - 3}{x - 2}$$ Substitute values.

$$\frac{1}{2}(x - 2) = y - 3$$ Multiply each side by $(x - 2)$.

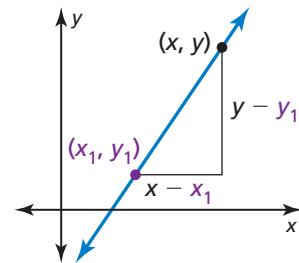The equation in *point-slope form* is $y - 3 = \frac{1}{2}(x - 2)$.

Core Concept

Point-Slope Form

Words   A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

slope

Algebra $y - y_1 = m(x - x_1)$

passes through (x_1, y_1)

EXAMPLE 1   Using a Slope and a Point to Write an Equation

Write an equation in point-slope form of the line that passes through the point $(-8, 3)$ and has a slope of $\frac{1}{4}$.

SOLUTION

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 3 = \frac{1}{4}[x - (-8)]$ Substitute $\frac{1}{4}$ for m, -8 for x_1, and 3 for y_1.

$y - 3 = \frac{1}{4}(x + 8)$ Simplify.

▶ The equation is $y - 3 = \frac{1}{4}(x + 8)$.

Check

$y - 3 = \frac{1}{4}(x + 8)$

$3 - 3 \overset{?}{=} \frac{1}{4}(-8 + 8)$

$0 = 0$ ✓

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Write an equation in point-slope form of the line that passes through the given point and has the given slope.

1. $(3, -1)$; $m = -2$

2. $(4, 0)$; $m = -\frac{2}{3}$

Writing Equations of Lines Given Two Points

When you are given two points on a line, you can write an equation of the line using the following steps.

Step 1 Find the slope of the line.

Step 2 Use the slope and one of the points to write an equation of the line in point-slope form.

ANOTHER WAY

You can use either of the given points to write an equation of the line.

Use $m = -2$ and $(3, -2)$.

$$y - (-2) = -2(x - 3)$$
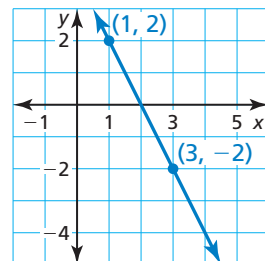$$y + 2 = -2x + 6$$
$$y = -2x + 4$$

EXAMPLE 2 Using Two Points to Write an Equation

Write an equation in slope-intercept form of the line shown.

SOLUTION

Step 1 Find the slope of the line.

$$m = \frac{-2 - 2}{3 - 1} = \frac{-4}{2}, \text{ or } -2$$

Step 2 Use the slope $m = -2$ and the point $(1, 2)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 2 = -2(x - 1)$	Substitute -2 for m, 1 for x_1, and 2 for y_1.
$y - 2 = -2x + 2$	Distributive Property
$y = -2x + 4$	Write in slope-intercept form.

▶ The equation is $y = -2x + 4$.

EXAMPLE 3 Writing a Linear Function

Write a linear function f with the values $f(4) = -2$ and $f(8) = 4$.

SOLUTION

Note that you can rewrite $f(4) = -2$ as $(4, -2)$ and $f(8) = 4$ as $(8, 4)$.

Step 1 Find the slope of the line that passes through $(4, -2)$ and $(8, 4)$.

$$m = \frac{4 - (-2)}{8 - 4} = \frac{6}{4}, \text{ or } 1.5$$

Step 2 Use the slope $m = 1.5$ and the point $(8, 4)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 4 = 1.5(x - 8)$	Substitute 1.5 for m, 8 for x_1, and 4 for y_1.
$y - 4 = 1.5x - 12$	Distributive Property
$y = 1.5x - 8$	Write in slope-intercept form.

▶ A function is $f(x) = 1.5x - 8$.

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Write an equation in slope-intercept form of the line that passes through the given points.

3. $(1, 4), (3, 10)$ **4.** $(-4, -1), (8, -4)$

5. Write a linear function g with the values $g(2) = 3$ and $g(6) = 5$.

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

The student council is ordering customized foam hands to promote school spirit. The table shows the cost of ordering different numbers of foam hands. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the cost as a function of the number of foam hands.

Number of foam hands	4	6	8	10	12
Cost (dollars)	34	46	58	70	82

SOLUTION

1. **Understand the Problem** You know five data pairs from the table. You are asked whether the data are linear. If so, write a linear model that represents the cost.

2. **Make a Plan** Find the rate of change for consecutive data pairs in the table. If the rate of change is constant, use the point-slope form to write an equation. Rewrite the equation in slope-intercept form so that the cost is a function of the number of foam hands.

3. **Solve the Problem**

   Step 1 Find the rate of change for consecutive data pairs in the table.

   $$\frac{46 - 34}{6 - 4} = 6, \frac{58 - 46}{8 - 6} = 6, \frac{70 - 58}{10 - 8} = 6, \frac{82 - 70}{12 - 10} = 6$$

   Because the rate of change is constant, the data are linear. So, use the point-slope form to write an equation that represents the data.

   Step 2 Use the constant rate of change (slope) $m = 6$ and the data pair $(4, 34)$ to write an equation. Let C be the cost (in dollars) and n be the number of foam hands.

$C - C_1 = m(n - n_1)$	Write the point-slope form.
$C - 34 = 6(n - 4)$	Substitute 6 for m, 4 for n_1, and 34 for C_1.
$C - 34 = 6n - 24$	Distributive Property
$C = 6n + 10$	Write in slope-intercept form.

   ▶ Because the cost increases at a constant rate, the situation can be modeled by a linear equation. The linear model is $C = 6n + 10$.

4. **Look Back** To check that your model is correct, verify that the other data pairs are solutions of the equation.

   $46 = 6(6) + 10$ ✓ $58 = 6(8) + 10$ ✓

   $70 = 6(10) + 10$ ✓ $82 = 6(12) + 10$ ✓

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6. You pay an installation fee and a monthly fee for Internet service. The table shows the total cost for different numbers of months. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the total cost as a function of the number of months.

| Number of months	Total cost (dollars)
3	176
6	302
9	428
12	554

Vocabulary and Core Concept Check

1. **USING STRUCTURE** Without simplifying, identify the slope of the line given by the equation $y - 5 = -2(x + 5)$. Then identify one point on the line.

2. **WRITING** Explain how you can use the slope formula to write an equation of the line that passes through $(3, -2)$ and has a slope of 4.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, write an equation in point-slope form of the line that passes through the given point and has the given slope. *(See Example 1.)*

3. $(2, 1)$; $m = 2$

4. $(3, 5)$; $m = -1$

5. $(7, -4)$; $m = -6$

6. $(-8, -2)$; $m = 5$

7. $(9, 0)$; $m = -3$

8. $(0, 2)$; $m = 4$

9. $(-6, 6)$; $m = \frac{3}{2}$

10. $(5, -12)$; $m = -\frac{2}{5}$

In Exercises 11–14, write an equation in slope-intercept form of the line shown. *(See Example 2.)*

11.

12.

13.

14.
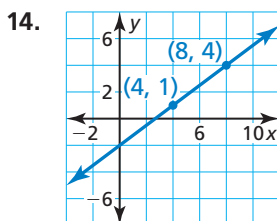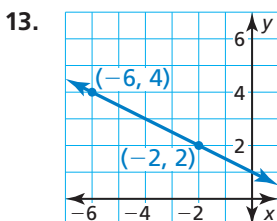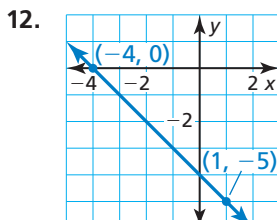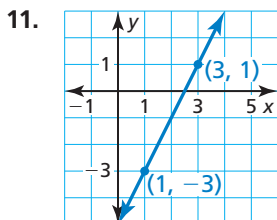
In Exercises 15–20, write an equation in slope-intercept form of the line that passes through the given points.

15. $(7, 2), (2, 12)$

16. $(6, -2), (12, 1)$

17. $(6, -1), (3, -7)$

18. $(-2, 5), (-4, -5)$

19. $(1, -9), (-3, -9)$

20. $(-5, 19), (5, 13)$

In Exercises 21–26, write a linear function f with the given values. *(See Example 3.)*

21. $f(2) = -2, f(1) = 1$

22. $f(5) = 7, f(-2) = 0$

23. $f(-4) = 2, f(6) = -3$

24. $f(-10) = 4, f(-2) = 4$

25. $f(-3) = 1, f(13) = 5$

26. $f(-9) = 10, f(-1) = -2$

In Exercises 27–30, tell whether the data in the table can be modeled by a linear equation. Explain. If possible, write a linear equation that represents y as a function of x. *(See Example 4.)*

27.

x	2	4	6	8	10
y	-1	5	15	29	47

28.

x	-3	-1	1	3	5
y	16	10	4	-2	-8

29.

x	y
0	1.2
1	1.4
2	1.6
4	2

30.

x	y
1	18
2	15
4	12
8	9

31. **ERROR ANALYSIS** Describe and correct the error in writing a linear function g with the values $g(5) = 4$ and $g(3) = 10$.

$$m = \frac{10 - 4}{3 - 5}$$
$$= \frac{6}{-2} = -3$$

$$y - y_1 = mx - x_1$$
$$y - 4 = -3x - 5$$
$$y = -3x - 1$$

A function is $g(x) = -3x - 1$.

32. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through the points (1, 2) and (4, 3).

✗ $m = \dfrac{3-2}{4-1} = \dfrac{1}{3}$ $y - 2 = \dfrac{1}{3}(x - 4)$

33. MODELING WITH MATHEMATICS You are designing a sticker to advertise your band. A company charges $225 for the first 1000 stickers and $80 for each additional 1000 stickers.

  a. Write an equation that represents the total cost (in dollars) of the stickers as a function of the number (in thousands) of stickers ordered.

  b. Find the total cost of 9000 stickers.

34. MODELING WITH MATHEMATICS You pay a processing fee and a daily fee to rent a beach house. The table shows the total cost of renting the beach house for different numbers of days.

Days	2	4	6	8
Total cost (dollars)	246	450	654	858

  a. Can the situation be modeled by a linear equation? Explain.

  b. What is the processing fee? the daily fee?

  c. You can spend no more than $1200 on the beach house rental. What is the maximum number of days you can rent the beach house?

35. WRITING Describe two ways to graph the equation $y - 1 = \dfrac{3}{2}(x - 4)$.

36. THOUGHT PROVOKING The graph of a linear function passes through the point $(12, -5)$ and has a slope of $\dfrac{2}{5}$. Represent this function in two other ways.

37. REASONING You are writing an equation of the line that passes through two points that are not on the y-axis. Would you use slope-intercept form or point-slope form to write the equation? Explain.

38. HOW DO YOU SEE IT? The graph shows two points that lie on the graph of a linear function.
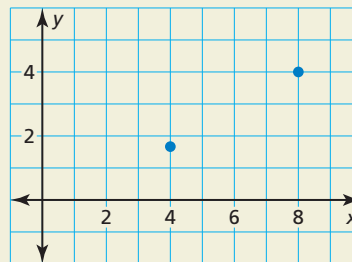
  a. Does the y-intercept of the graph of the linear function appear to be positive or negative? Explain.

  b. Estimate the coordinates of the two points. How can you use your estimates to confirm your answer in part (a)?

39. CONNECTION TO TRANSFORMATIONS Compare the graph of $y = 2x$ to the graph of $y - 1 = 2(x + 3)$. Make a conjecture about the graphs of $y = mx$ and $y - k = m(x - h)$.

40. COMPARING FUNCTIONS Three siblings each receive money for a holiday and then spend it at a constant weekly rate. The graph describes Sibling A's spending, the table describes Sibling B's spending, and the equation $y = -22.5x + 90$ describes Sibling C's spending. The variable y represents the amount of money left after x weeks.
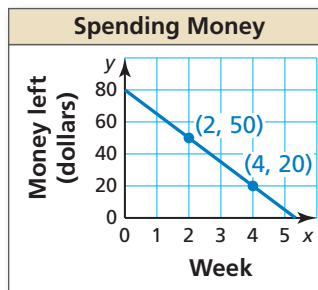
Week, x	Money left, y
1	$100
2	$75
3	$50
4	$25

  a. Which sibling received the most money? the least money?

  b. Which sibling spends money at the fastest rate? the slowest rate?

  c. Which sibling runs out of money first? last?

Maintaining Mathematical Proficiency   Reviewing what you learned in previous grades and lessons

Write the reciprocal of the number. *(Skills Review Handbook)*

41. 5   **42.** -8 **43.** $-\dfrac{2}{7}$ **44.** $\dfrac{3}{2}$