5.3 Solving Systems of Linear Equations by Elimination

Essential Question

How can you use elimination to solve a system of linear equations?

EXPLORATION 1 Writing and Solving a System of Equations

Work with a partner. You purchase a drink and a sandwich for \$4.50. Your friend purchases a drink and five sandwiches for \$16.50. You want to determine the price of a drink and the price of a sandwich.

a. Let x represent the price (in dollars) of one drink. Let y represent the price (in dollars) of one sandwich. Write a system of equations for the situation. Use the following verbal model.

Number of drinks • Price per drink + Number of sandwiches • Price per sandwich = Total price

Label one of the equations Equation 1 and the other equation Equation 2.

b. Subtract Equation 1 from Equation 2. Explain how you can use the result to solve the system of equations. Then find and interpret the solution.

CHANGING COURSE

To be proficient in math, you need to monitor and evaluate your progress and change course using a different solution method, if necessary.

EXPLORATION 2 Using Elimination to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1 Subtract. Subtract Equation 2 from Equation 1. Then use the result to solve the system.

Method 2 Add. Add the two equations. Then use the result to solve the system.

Is the solution the same using both methods? Which method do you prefer?

a. $3x - y = 6$
$3x + y = 0$

b. $2x + y = 6$
$2x - y = 2$

c. $x - 2y = -7$
$x + 2y = 5$

EXPLORATION 3 Using Elimination to Solve a System

Work with a partner.

$2x + y = 7$ Equation 1
$x + 5y = 17$ Equation 2

a. Can you eliminate a variable by adding or subtracting the equations as they are? If not, what do you need to do to one or both equations so that you can?

b. Solve the system individually. Then exchange solutions with your partner and compare and check the solutions.

Communicate Your Answer

4. How can you use elimination to solve a system of linear equations?
5. When can you add or subtract the equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.
6. In Exploration 3, why can you multiply an equation in the system by a constant and not change the solution of the system? Explain your reasoning.

5.3 Lesson

Core Vocabulary

Previous
coefficient

What You Will Learn

- Solve systems of linear equations by elimination.
- Use systems of linear equations to solve real-life problems.

Solving Linear Systems by Elimination

Core Concept

Solving a System of Linear Equations by Elimination

Step 1 Multiply, if necessary, one or both equations by a constant so at least one pair of like terms has the same or opposite coefficients.

Step 2 Add or subtract the equations to eliminate one of the variables.

Step 3 Solve the resulting equation.

Step 4 Substitute the value from Step 3 into one of the original equations and solve for the other variable.

You can use elimination to solve a system of equations because replacing one equation in the system with the sum of that equation and a multiple of the other produces a system that has the same solution. Here is why.

System 1

$a = b$ Equation 1
$c = d$ Equation 2

System 2

$a + kc = b + kd$ Equation 3
$c = d$ Equation 2

Consider System 1. In this system, a and c are algebraic expressions, and b and d are constants. Begin by multiplying each side of Equation 2 by a constant k. By the Multiplication Property of Equality, $kc = kd$. You can rewrite Equation 1 as Equation 3 by adding kc on the left and kd on the right. You can rewrite Equation 3 as Equation 1 by subtracting kc on the left and kd on the right. Because you can rewrite either system as the other, System 1 and System 2 have the same solution.

EXAMPLE 1 Solving a System of Linear Equations by Elimination

Solve the system of linear equations by elimination.

$3x + 2y = 4$ Equation 1
$3x - 2y = -4$ Equation 2

SOLUTION

Step 1 Because the coefficients of the y-terms are opposites, you do not need to multiply either equation by a constant.

Step 2 Add the equations.

$3x + 2y = 4$ Equation 1
$\underline{3x - 2y = -4}$ Equation 2
$6x \quad = 0$ Add the equations.

Step 3 Solve for x.

$6x = 0$ Resulting equation from Step 2
$x = 0$ Divide each side by 6.

Step 4 Substitute 0 for x in one of the original equations and solve for y.

$3x + 2y = 4$ Equation 1
$3(0) + 2y = 4$ Substitute 0 for x.
$y = 2$ Solve for y.

- The solution is (0, 2).

Check

Equation 1

$3x + 2y = 4$
$3(0) + 2(2) \stackrel{?}{=} 4$
$4 = 4$ ✓

Equation 2

$3x - 2y = -4$
$3(0) - 2(2) \stackrel{?}{=} -4$
$-4 = -4$ ✓

EXAMPLE 2 Solving a System of Linear Equations by Elimination

Solve the system of linear equations by elimination.

$-10x + 3y = 1$ Equation 1

$-5x - 6y = 23$ Equation 2

ANOTHER WAY

To use subtraction to eliminate one of the variables, multiply Equation 2 by 2 and then subtract the equations.

$$\begin{array}{r} -10x + 3y = 1 \\ \underline{-(-10x - 12y = 46)} \\ 15y = -45 \end{array}$$

SOLUTION

Step 1 Multiply Equation 2 by -2 so that the coefficients of the x-terms are opposites.

$-10x + 3y = 1$ → $-10x + 3y = 1$ Equation 1

$-5x - 6y = 23$ **Multiply by –2.** → $10x + 12y = -46$ Revised Equation 2

Step 2 Add the equations.

$$\begin{array}{r} -10x + 3y = 1 \\ \underline{10x + 12y = -46} \\ 15y = -45 \end{array}$$

Equation 1
Revised Equation 2
Add the equations.

Step 3 Solve for y.

$15y = -45$ Resulting equation from Step 2

$y = -3$ Divide each side by 15.

Step 4 Substitute -3 for y in one of the original equations and solve for x.

$-5x - 6y = 23$ Equation 2

$-5x - 6(-3) = 23$ Substitute -3 for y.

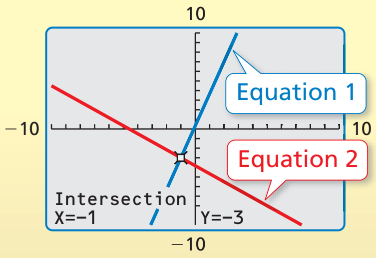
$-5x + 18 = 23$ Multiply.

$-5x = 5$ Subtract 18 from each side.

$x = -1$ Divide each side by -5.

▶ The solution is $(-1, -3)$.

Check



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by elimination. Check your solution.

1. $3x + 2y = 7$
$-3x + 4y = 5$

2. $x - 3y = 24$
$3x + y = 12$

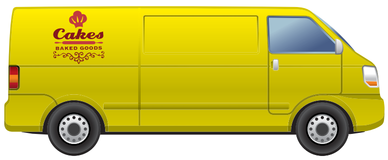
3. $x + 4y = 22$
$4x + y = 13$

Concept Summary

Methods for Solving Systems of Linear Equations

Method	When to Use
Graphing *(Lesson 5.1)*	To estimate solutions
Substitution *(Lesson 5.2)*	When one of the variables in one of the equations has a coefficient of 1 or -1
Elimination *(Lesson 5.3)*	When at least one pair of like terms has the same or opposite coefficients
Elimination (Multiply First) *(Lesson 5.3)*	When one of the variables cannot be eliminated by adding or subtracting the equations

Solving Real-Life Problems



EXAMPLE 3 Modeling with Mathematics

A business with two locations buys seven large delivery vans and five small delivery vans. Location A receives five large vans and two small vans for a total cost of \$235,000. Location B receives two large vans and three small vans for a total cost of \$160,000. What is the cost of each type of van?

SOLUTION

1. **Understand the Problem** You know how many of each type of van each location receives. You also know the total cost of the vans for each location. You are asked to find the cost of each type of van.
2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
3. **Solve the Problem**

Words $5 \cdot$ [Cost of large van] $+ 2 \cdot$ [Cost of small van] $= 235{,}000$

$2 \cdot$ [Cost of large van] $+ 3 \cdot$ [Cost of small van] $= 160{,}000$

Variables Let x be the cost (in dollars) of a large van and let y be the cost (in dollars) of a small van.

System $5x + 2y = 235{,}000$ Equation 1

$2x + 3y = 160{,}000$ Equation 2

STUDY TIP

In Example 3, both equations are multiplied by a constant so that the coefficients of the *y*-terms are opposites.

Step 1 Multiply Equation 1 by −3. Multiply Equation 2 by 2.

$5x + 2y = 235{,}000$ Multiply by −3. → $-15x - 6y = -705{,}000$ Revised Equation 1

$2x + 3y = 160{,}000$ Multiply by 2. → $4x + 6y = 320{,}000$ Revised Equation 2

Step 2 Add the equations.

$-15x - 6y = -705{,}000$ Revised Equation 1

$4x + 6y = 320{,}000$ Revised Equation 2

$-11x = -385{,}000$ Add the equations.

Step 3 Solving the equation $-11x = -385{,}000$ gives $x = 35{,}000$.

Step 4 Substitute 35,000 for x in one of the original equations and solve for y.

$5x + 2y = 235{,}000$ Equation 1

$5(35{,}000) + 2y = 235{,}000$ Substitute 35,000 for *x*.

$y = 30{,}000$ Solve for *y*.

▶ The solution is (35,000, 30,000). So, a large van costs \$35,000 and a small van costs \$30,000.

4. **Look Back** Check to make sure your solution makes sense with the given information. For Location A, the total cost is $5(35{,}000) + 2(30{,}000) = \$235{,}000$. For Location B, the total cost is $2(35{,}000) + 3(30{,}000) = \$160{,}000$. So, the solution makes sense.

Monitoring Progress

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4. Solve the system in Example 3 by eliminating x.

5.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Give an example of a system of linear equations that can be solved by first adding the equations to eliminate one variable.

2. **WRITING** Explain how to solve the system of linear equations by elimination.

$2x - 3y = -4$ Equation 1
$-5x + 9y = 7$ Equation 2

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the system of linear equations by elimination. Check your solution. *(See Example 1.)*

3. $x + 2y = 13$
$-x + y = 5$

4. $9x + y = 2$
$-4x - y = -17$

5. $5x + 6y = 50$
$x - 6y = -26$

6. $-x + y = 4$
$x + 3y = 4$

7. $-3x - 5y = -7$
$-4x + 5y = 14$

8. $4x - 9y = -21$
$-4x - 3y = 9$

9. $-y - 10 = 6x$
$5x + y = -10$

10. $3x - 30 = y$
$7y - 6 = 3x$

In Exercises 11–18, solve the system of linear equations by elimination. Check your solution. *(See Examples 2 and 3.)*

11. $x + y = 2$
$2x + 7y = 9$

12. $8x - 5y = 11$
$4x - 3y = 5$

13. $11x - 20y = 28$
$3x + 4y = 36$

14. $10x - 9y = 46$
$-2x + 3y = 10$

15. $4x - 3y = 8$
$5x - 2y = -11$

16. $-2x - 5y = 9$
$3x + 11y = 4$

17. $9x + 2y = 39$
$6x + 13y = -9$

18. $12x - 7y = -2$
$8x + 11y = 30$

19. ERROR ANALYSIS Describe and correct the error in solving for one of the variables in the linear system $5x - 7y = 16$ and $x + 7y = 8$.

✗

$$\begin{aligned} 5x - 7y &= 16 \\ x + 7y &= 8 \\ \hline 4x \quad &= 24 \\ x &= 6 \end{aligned}$$

20. ERROR ANALYSIS Describe and correct the error in solving for one of the variables in the linear system $4x + 3y = 8$ and $x - 2y = -13$.

✗

$4x + 3y = 8$
$x - 2y = -13$ **Multiply by −4.**

$$\begin{aligned} 4x + 3y &= 8 \\ -4x + 8y &= -13 \\ \hline 11y &= -5 \\ y &= \frac{-5}{11} \end{aligned}$$

21. MODELING WITH MATHEMATICS A service center charges a fee of x dollars for an oil change plus y dollars per quart of oil used. A sample of its sales record is shown. Write a system of linear equations that represents this situation. Find the fee and cost per quart of oil.

	A	B	C
1	Customer	Oil Tank Size (quarts)	Total Cost
2	A	5	$22.45
3	B	7	$25.45
4			

22. MODELING WITH MATHEMATICS A music website charges x dollars for individual songs and y dollars for entire albums. Person A pays $25.92 to download 6 individual songs and 2 albums. Person B pays $33.93 to download 4 individual songs and 3 albums. Write a system of linear equations that represents this situation. How much does the website charge to download a song? an entire album?

In Exercises 23–26, solve the system of linear equations using any method. Explain why you chose the method.

23. $3x + 2y = 4$
$2y = 8 - 5x$

24. $-6y + 2 = -4x$
$y - 2 = x$

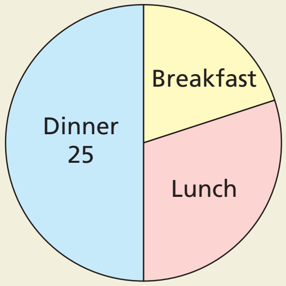
25. $y - x = 2$
$y = -\frac{1}{4}x + 7$

26. $3x + y = \frac{1}{3}$
$2x - 3y = \frac{8}{3}$

27. WRITING For what values of a can you solve the linear system $ax + 3y = 2$ and $4x + 5y = 6$ by elimination without multiplying first? Explain.

28. HOW DO YOU SEE IT? The circle graph shows the results of a survey in which 50 students were asked about their favorite meal.

Favorite Meal



a. Estimate the numbers of students who chose breakfast and lunch.

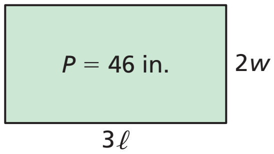
b. The number of students who chose lunch was 5 more than the number of students who chose breakfast. Write a system of linear equations that represents the numbers of students who chose breakfast and lunch.

c. Explain how you can solve the linear system in part (b) to check your answers in part (a).

29. MAKING AN ARGUMENT Your friend says that any system of equations that can be solved by elimination can be solved by substitution in an equal or fewer number of steps. Is your friend correct? Explain.

30. THOUGHT PROVOKING Write a system of linear equations that can be added to eliminate a variable *or* subtracted to eliminate a variable.

31. MATHEMATICAL CONNECTIONS A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width w and tripling the length ℓ, as shown. The new rectangle has a perimeter P of 46 inches.



a. Write and solve a system of linear equations to find the length and width of the original rectangle.

b. Find the length and width of the new rectangle.

32. CRITICAL THINKING Refer to the discussion of System 1 and System 2 on page 248. Without solving, explain why the two systems shown have the same solution.

System 1		System 2	
$3x - 2y = 8$	Equation 1	$5x = 20$	Equation 3
$x + y = 6$	Equation 2	$x + y = 6$	Equation 2

33. PROBLEM SOLVING You are making 6 quarts of fruit punch for a party. You have bottles of 100% fruit juice and 20% fruit juice. How many quarts of each type of juice should you mix to make 6 quarts of 80% fruit juice?

34. PROBLEM SOLVING A motorboat takes 40 minutes to travel 20 miles downstream. The return trip takes 60 minutes. What is the speed of the current?

35. CRITICAL THINKING Solve for x, y, and z in the system of equations. Explain your steps.

$x + 7y + 3z = 29$ Equation 1
$3z + x - 2y = -7$ Equation 2
$5y = 10 - 2x$ Equation 3

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. *(Section 1.3)*

36. $5d - 8 = 1 + 5d$

37. $9 + 4t = 12 - 4t$

38. $3n + 2 = 2(n - 3)$

39. $-3(4 - 2v) = 6v - 12$

Write an equation of the line that passes through the given point and is parallel to the given line. *(Section 4.3)*

40. $(4, -1)$; $y = -2x + 7$

41. $(0, 6)$; $y = 5x - 3$

42. $(-5, -2)$; $y = \frac{2}{3}x + 1$