10.1 Graphing Square Root Functions

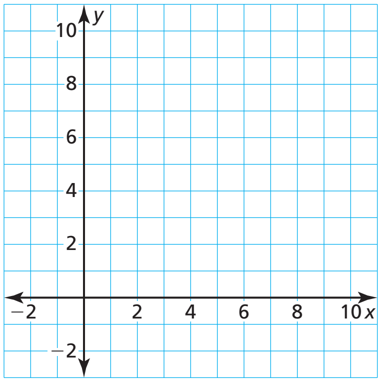
Essential Question What are some of the characteristics of the graph of a square root function?

EXPLORATION 1 Graphing Square Root Functions

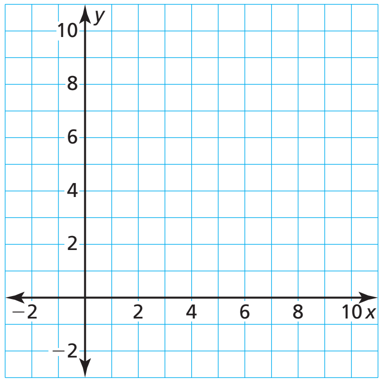
Work with a partner.

- Make a table of values for each function.
- Use the table to sketch the graph of each function.
- Describe the domain of each function.
- Describe the range of each function.

a. $y = \sqrt{x}$



b. $y = \sqrt{x + 2}$



EXPLORATION 2 Writing Square Root Functions

LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern a pattern or structure.

Work with a partner. Write a square root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

x	$f(x)$
−4	0
−3	
−2	
−1	$\sqrt{3}$
0	2
1	

b.

x	$f(x)$
−4	1
−3	
−2	
−1	$1 + \sqrt{3}$
0	3
1	

Communicate Your Answer

3. What are some of the characteristics of the graph of a square root function?

4. Graph each function. Then compare the graph to the graph of $f(x) = \sqrt{x}$.

a. $g(x) = \sqrt{x - 1}$ **b.** $g(x) = \sqrt{x} - 1$ **c.** $g(x) = 2\sqrt{x}$ **d.** $g(x) = -2\sqrt{x}$

10.1 Lesson

Core Vocabulary

square root function, *p. 544*
radical function, *p. 545*

Previous
radicand
transformation
average rate of change

What You Will Learn

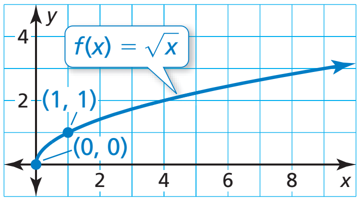
- Graph square root functions.
- Compare square root functions using average rates of change.
- Solve real-life problems involving square root functions.

Graphing Square Root Functions

Core Concept

Square Root Functions

A **square root function** is a function that contains a square root with the independent variable in the radicand. The parent function for the family of square root functions is $f(x) = \sqrt{x}$. The domain of f is $x \geq 0$, and the range of f is $y \geq 0$.



STUDY TIP

The graph of $f(x) = \sqrt{x}$ starts at (0, 0) and increases on the entire domain. So, the minimum value of f is 0.

The value of the radicand in a square root function cannot be negative. So, the domain of a square root function includes x-values for which the radicand is greater than or equal to 0.

EXAMPLE 1 Describing the Domain of a Square Root Function

Describe the domain of $f(x) = 3\sqrt{x - 5}$.

SOLUTION

The radicand cannot be negative. So, $x - 5$ is greater than or equal to 0.

$x - 5 \geq 0$ — Write an inequality for the domain.

$x \geq 5$ — Add 5 to each side.

▶ The domain is the set of real numbers greater than or equal to 5.

EXAMPLE 2 Graphing a Square Root Function

Graph $f(x) = \sqrt{x} + 3$. Describe the range of the function.

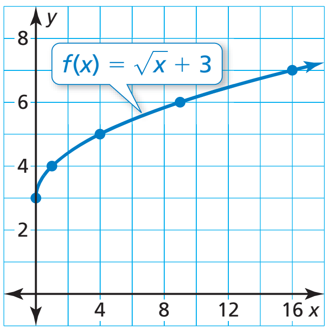
SOLUTION

Step 1 Use the domain of f, $x \geq 0$, to make a table of values.

x	0	1	4	9	16
$f(x)$	3	4	5	6	7

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points, starting at (0, 3).



▶ From the graph, you can see that the range of f is $y \geq 3$.

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Describe the domain of the function.

1. $f(x) = 10\sqrt{x}$ **2.** $y = \sqrt{2x} + 7$ **3.** $h(x) = \sqrt{-x + 1}$

Graph the function. Describe the range.

4. $g(x) = \sqrt{x} - 4$ **5.** $y = \sqrt{x + 5}$ **6.** $n(x) = 5\sqrt{x}$

STUDY TIP

You will study another type of radical function in the next section.

A **radical function** is a function that contains a radical expression with the independent variable in the radicand. A square root function is a type of radical function.

You can transform graphs of radical functions in the same way you transformed graphs of functions previously. In Example 2, notice that the graph of f is a vertical translation of the graph of the parent square root function.

Core Concept

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = \sqrt{x - 2}$ $g(x) = \sqrt{x + 3}$	2 units right 3 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = \sqrt{x} + 7$ $g(x) = \sqrt{x} - 1$	7 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \sqrt{-x}$ $g(x) = -\sqrt{x}$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = \sqrt{3x}$ $g(x) = \sqrt{\frac{1}{2}x}$	shrink by a factor of $\frac{1}{3}$ stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$	$g(x) = 4\sqrt{x}$ $g(x) = \frac{1}{5}\sqrt{x}$	stretch by a factor of 4 shrink by a factor of $\frac{1}{5}$

EXAMPLE 3 Comparing Graphs of Square Root Functions

Graph $g(x) = -\sqrt{x - 2}$. Compare the graph to the graph of $f(x) = \sqrt{x}$.

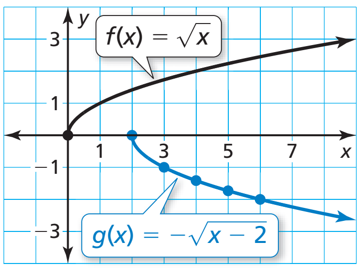
SOLUTION

Step 1 Use the domain of g, $x \geq 2$, to make a table of values.

x	2	3	4	5	6
$g(x)$	0	−1	−1.4	−1.7	−2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points, starting at (2, 0).



The graph of g is a translation 2 units right and a reflection in the x-axis of the graph of f.

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Graph the function. Compare the graph to the graph of $f(x) = \sqrt{x}$.

7. $h(x) = \sqrt{\frac{1}{4}x}$ **8.** $g(x) = \sqrt{x} - 6$ **9.** $m(x) = -3\sqrt{x}$

To graph a square root function of the form $y = a\sqrt{x - h} + k$, where $a \neq 0$, start at (h, k).

EXAMPLE 4 **Graphing $y = a\sqrt{x - h} + k$**

REMEMBER
The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

Let $g(x) = -2\sqrt{x - 3} - 2$. (a) Describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of g. (b) Graph g.

SOLUTION

a. Step 1 Translate the graph of f horizontally 3 units right to get the graph of $t(x) = \sqrt{x - 3}$.

Step 2 Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2\sqrt{x - 3}$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2\sqrt{x - 3}$.

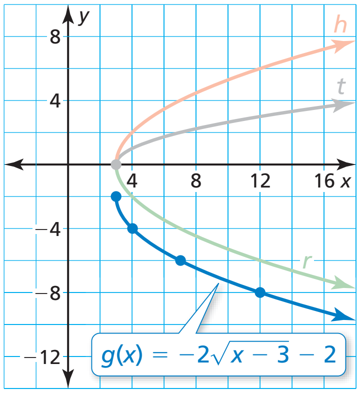
Step 4 Translate the graph of r vertically 2 units down to get the graph of $g(x) = -2\sqrt{x - 3} - 2$.

b. Step 1 Use the domain, $x \geq 3$, to make a table of values.

x	3	4	7	12
$g(x)$	-2	-4	-6	-8

Step 2 Plot the ordered pairs.

Step 3 Start at $(h, k) = (3, -2)$ and draw a smooth curve through the points.



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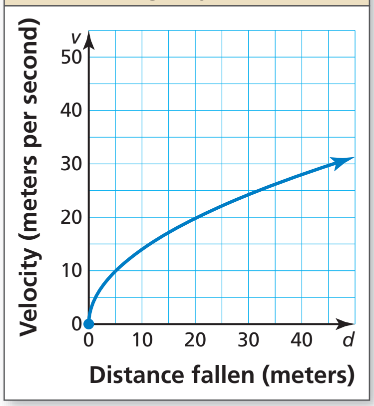
10. Let $g(x) = \frac{1}{2}\sqrt{x + 4} + 1$. Describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of g. Then graph g.

Comparing Average Rates of Change

EXAMPLE 5 **Comparing Square Root Functions**

The model $v(d) = \sqrt{2gd}$ represents the velocity v (in meters per second) of a free-falling object on the moon, where g is the constant 1.6 meters per second squared and d is the distance (in meters) the object has fallen. The velocity of a free-falling object on Earth is shown in the graph. Compare the velocities by finding and interpreting their average rates of change over the interval $d = 0$ to $d = 10$.

Free-Falling Object on Earth



SOLUTION

To calculate the average rates of change, use points whose d-coordinates are 0 and 10.

Earth: Use the graph to estimate. Use (0, 0) and (10, 14).

$$\frac{v(10) - v(0)}{10 - 0} \approx \frac{14 - 0}{10} = 1.4 \qquad \text{Average rate of change on Earth}$$

Moon: Evaluate v when $d = 0$ and $d = 10$.

$$v(0) = \sqrt{2(1.6)(0)} = 0 \quad \text{and} \quad v(10) = \sqrt{2(1.6)(10)} = \sqrt{32} \approx 5.7$$

Use $(0, 0)$ and $(10, \sqrt{32})$.

$$\frac{v(10) - v(0)}{10 - 0} = \frac{\sqrt{32} - 0}{10} \approx 0.57 \qquad \text{Average rate of change on the moon}$$

▶ From 0 to 10 meters, the velocity of a free-falling object increases at an average rate of about 1.4 meters per second per meter on Earth and about 0.57 meter per second per meter on the moon.

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11. In Example 5, compare the velocities by finding and interpreting their average rates of change over the interval $d = 30$ to $d = 40$.

Solving Real-Life Problems

EXAMPLE 6 **Real-Life Application**

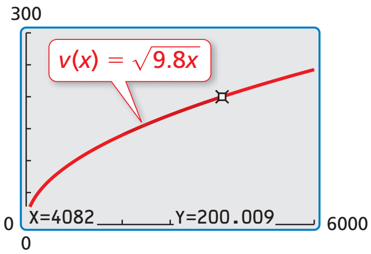
The velocity v (in meters per second) of a tsunami can be modeled by the function $v(x) = \sqrt{9.8x}$, where x is the water depth (in meters). (a) Use a graphing calculator to graph the function. At what depth does the velocity of the tsunami exceed 200 meters per second? (b) What happens to the average rate of change of the velocity as the water depth increases?

SOLUTION

1. **Understand the Problem** You know the function that models the velocity of a tsunami based on water depth. You are asked to graph the function using a calculator and find the water depth where the velocity exceeds 200 meters per second. Then you are asked to describe the average rate of change of the velocity as the water depth increases.

2. **Make a Plan** Graph the function using a calculator. Use the *trace* feature to find the value of x when $v(x)$ is about 200. Then calculate and compare average rates of change of the velocity over different intervals.

3. **Solve the Problem**



a. Step 1 Enter the function into your calculator and graph it.

Step 2 Use the *trace* feature to find the value of x when $v(x) \approx 200$.

▶ The velocity exceeds 200 meters per second at a depth of about 4082 meters.

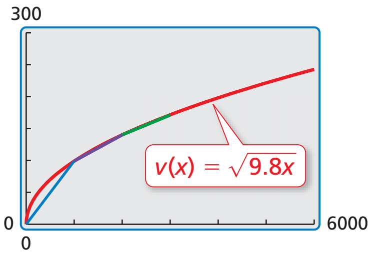
b. Calculate the average rates of change over the intervals $x = 0$ to $x = 1000$, $x = 1000$ to $x = 2000$, and $x = 2000$ to $x = 3000$.

$$\frac{v(1000) - v(0)}{1000 - 0} = \frac{\sqrt{9800} - 0}{1000} \approx 0.099 \qquad \text{0 to 1000 meters}$$

$$\frac{v(2000) - v(1000)}{2000 - 1000} = \frac{\sqrt{19{,}600} - \sqrt{9800}}{1000} \approx 0.041 \qquad \text{1000 to 2000 meters}$$

$$\frac{v(3000) - v(2000)}{3000 - 2000} = \frac{\sqrt{29{,}400} - \sqrt{19{,}600}}{1000} \approx 0.031 \qquad \text{2000 to 3000 meters}$$

▶ The average rate of change of the velocity decreases as the water depth increases.



4. **Look Back** To check the answer in part (a), find $v(x)$ when $x = 4082$.

$$v(4082) = \sqrt{9.8(4082)} \approx 200 \ \checkmark$$

In part (b), the slopes of the line segments (shown at the left) that represent the average rates of change over the intervals are decreasing. So, the answer to part (b) is reasonable.

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12. **WHAT IF?** At what depth does the velocity of the tsunami exceed 100 meters per second?

10.1 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A ________ is a function that contains a radical expression with the independent variable in the radicand.

2. **VOCABULARY** Is $y = 2x\sqrt{5}$ a square root function? Explain.

3. **WRITING** How do you describe the domain of a square root function?

4. **REASONING** Is the graph of $g(x) = 1.25\sqrt{x}$ a vertical stretch or a vertical shrink of the graph of $f(x) = \sqrt{x}$? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–14, describe the domain of the function. *(See Example 1.)*

5. $y = 8\sqrt{x}$

6. $y = \sqrt{4x}$

7. $y = 4 + \sqrt{-x}$

8. $y = \sqrt{-\frac{1}{2}x} + 1$

9. $h(x) = \sqrt{x - 4}$

10. $p(x) = \sqrt{x + 7}$

11. $f(x) = \sqrt{-x + 8}$

12. $g(x) = \sqrt{-x - 1}$

13. $m(x) = 2\sqrt{x + 4}$

14. $n(x) = \frac{1}{2}\sqrt{-x - 2}$

In Exercises 15–18, match the function with its graph. Describe the range.

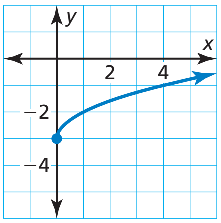
15. $y = \sqrt{x - 3}$

16. $y = 3\sqrt{x}$

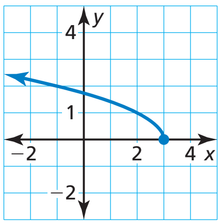
17. $y = \sqrt{x} - 3$

18. $y = \sqrt{-x + 3}$

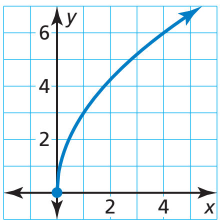
A.



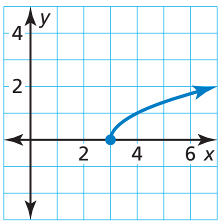
B.



C.



D.



In Exercises 19–26, graph the function. Describe the range. *(See Example 2.)*

19. $y = \sqrt{3x}$

20. $y = 4\sqrt{-x}$

21. $y = \sqrt{x} + 5$

22. $y = -2 + \sqrt{x}$

23. $f(x) = -\sqrt{x - 3}$

24. $g(x) = \sqrt{x + 4}$

25. $h(x) = \sqrt{x + 2} - 2$

26. $f(x) = -\sqrt{x - 1} + 3$

In Exercises 27–34, graph the function. Compare the graph to the graph of $f(x) = \sqrt{x}$. *(See Example 3.)*

27. $g(x) = \frac{1}{4}\sqrt{x}$

28. $r(x) = \sqrt{2x}$

29. $h(x) = \sqrt{x + 3}$

30. $q(x) = \sqrt{x} + 8$

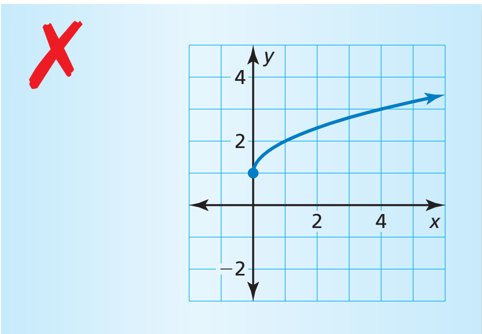
31. $p(x) = \sqrt{-\frac{1}{3}x}$

32. $g(x) = -5\sqrt{x}$

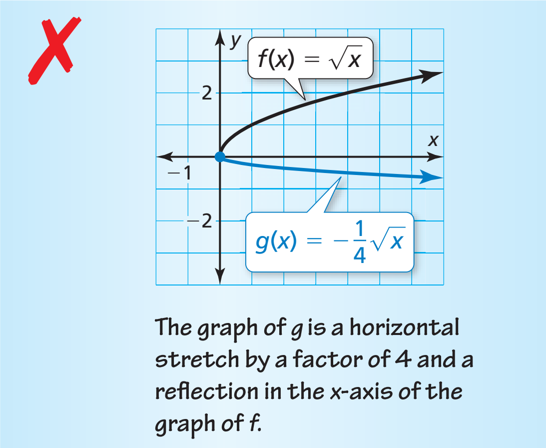
33. $m(x) = -\sqrt{x} - 6$

34. $n(x) = -\sqrt{x - 4}$

35. **ERROR ANALYSIS** Describe and correct the error in graphing the function $y = \sqrt{x + 1}$.



36. ERROR ANALYSIS Describe and correct the error in comparing the graph of $g(x) = -\frac{1}{4}\sqrt{x}$ to the graph of $f(x) = \sqrt{x}$.



In Exercises 37–44, describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of h. Then graph h. *(See Example 4.)*

37. $h(x) = 4\sqrt{x+2} - 1$

38. $h(x) = \frac{1}{2}\sqrt{x-6} + 3$

39. $h(x) = 2\sqrt{-x} - 6$

40. $h(x) = -\sqrt{x-3} - 2$

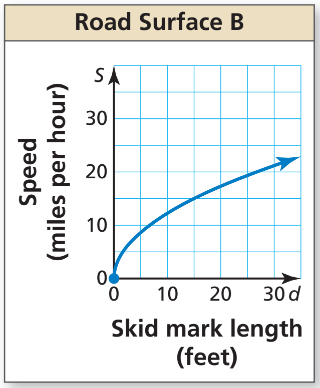
41. $h(x) = \frac{1}{3}\sqrt{x+3} - 3$

42. $h(x) = 2\sqrt{x-1} + 4$

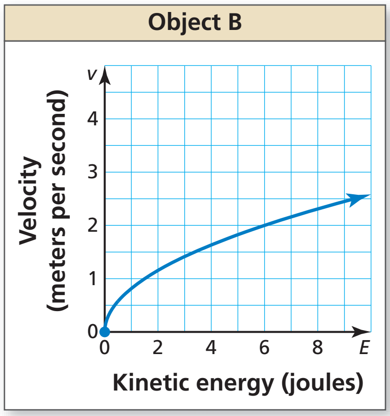
43. $h(x) = -2\sqrt{x-1} + 5$

44. $h(x) = -5\sqrt{x+2} - 1$

45. COMPARING FUNCTIONS The model $S(d) = \sqrt{30df}$ represents the speed S (in miles per hour) of a van before it skids to a stop, where f is the drag factor of the road surface and d is the length (in feet) of the skid marks. The drag factor of Road Surface A is 0.75. The graph shows the speed of the van on Road Surface B. Compare the speeds by finding and interpreting their average rates of change over the interval $d = 0$ to $d = 15$. *(See Example 5.)*



46. COMPARING FUNCTIONS The velocity v (in meters per second) of an object in motion is given by $v(E) = \sqrt{\frac{2E}{m}}$, where E is the kinetic energy of the object (in joules) and m is the mass of the object (in kilograms). The mass of Object A is 4 kilograms. The graph shows the velocity of Object B. Compare the velocities of the objects by finding and interpreting the average rates of change over the interval $E = 0$ to $E = 6$.



47. OPEN-ENDED Consider the graph of $y = \sqrt{x}$.

a. Write a function that is a vertical translation of the graph of $y = \sqrt{x}$.

b. Write a function that is a reflection of the graph of $y = \sqrt{x}$.

48. REASONING Can the domain of a square root function include negative numbers? Can the range include negative numbers? Explain your reasoning.

49. PROBLEM SOLVING The nozzle pressure of a fire hose allows firefighters to control the amount of water they spray on a fire. The flow rate f (in gallons per minute) can be modeled by the function $f = 120\sqrt{p}$, where p is the nozzle pressure (in pounds per square inch). *(See Example 6.)*

a. Use a graphing calculator to graph the function. At what pressure does the flow rate exceed 300 gallons per minute?

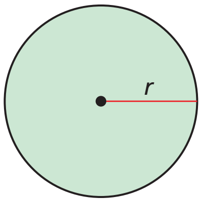
b. What happens to the average rate of change of the flow rate as the pressure increases?

50. **PROBLEM SOLVING** The speed s (in meters per second) of a long jumper before jumping can be modeled by the function $s = 10.9\sqrt{h}$, where h is the maximum height (in meters from the ground) of the jumper.

a. Use a graphing calculator to graph the function. A jumper is running 9.2 meters per second. Estimate the maximum height of the jumper.

b. Suppose the runway and pit are raised on a platform slightly higher than the ground. How would the graph of the function be transformed?

51. **MATHEMATICAL CONNECTIONS** The radius r of a circle is given by $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the circle.



a. Describe the domain of the function. Use a graphing calculator to graph the function.

b. Use the *trace* feature to approximate the area of a circle with a radius of 5.4 inches.

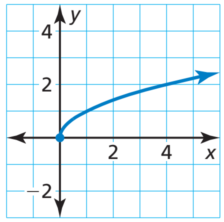
52. **REASONING** Consider the function $f(x) = 8a\sqrt{x}$.

a. For what value of a will the graph of f be identical to the graph of the parent square root function?

b. For what values of a will the graph of f be a vertical stretch of the graph of the parent square root function?

c. For what values of a will the graph of f be a vertical shrink and a reflection of the graph of the parent square root function?

53. **REASONING** The graph represents the function $f(x) = \sqrt{x}$.



a. What is the minimum value of the function?

b. Does the function have a maximum value? Explain.

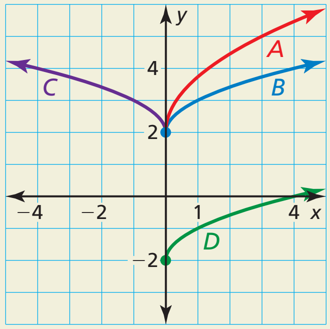
c. Write a square root function that has a maximum value. Does the function have a minimum value? Explain.

d. Write a square root function that has a minimum value of -4.

54. **HOW DO YOU SEE IT?** Match each function with its graph. Explain your reasoning.

A. $f(x) = \sqrt{x} + 2$ **B.** $m(x) = f(x) - 4$

C. $n(x) = f(-x)$ **D.** $p(x) = f(3x)$



55. **REASONING** Without graphing, determine which function's graph rises more steeply, $f(x) = 5\sqrt{x}$ or $g(x) = \sqrt{5x}$. Explain your reasoning.

56. **THOUGHT PROVOKING** Use a graphical approach to find the solutions of $x - 1 = \sqrt{5x - 9}$. Show your work. Verify your solutions algebraically.

57. **OPEN-ENDED** Write a radical function that has a domain of all real numbers greater than or equal to -5 and a range of all real numbers less than or equal to 3.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Section 6.2)*

58. $\sqrt[3]{343}$ **59.** $\sqrt[3]{-64}$ **60.** $-\sqrt[3]{-\frac{1}{27}}$

Factor the polynomial. *(Section 7.5)*

61. $x^2 + 7x + 6$ **62.** $d^2 - 11d + 28$ **63.** $y^2 - 3y - 40$