7.2 Multiplying Polynomials

Essential Question

How can you multiply two polynomials?

EXPLORATION 1 Multiplying Monomials Using Algebra Tiles

Work with a partner. Write each product. Explain your reasoning.

a. $(+1) \cdot (+1) =$ ____

b. $(+1) \cdot (-1) =$ ____

c. $(-1) \cdot (-1) =$ ____

d. $(+x) \cdot (+1) =$ ____

e. $(+x) \cdot (-1) =$ ____

f. $(-x) \cdot (+1) =$ ____

g. $(-x) \cdot (-1) =$ ____

h. $(+x) \cdot (+x) =$ ____

i. $(+x) \cdot (-x) =$ ____

j. $(-x) \cdot (-x) =$ ____

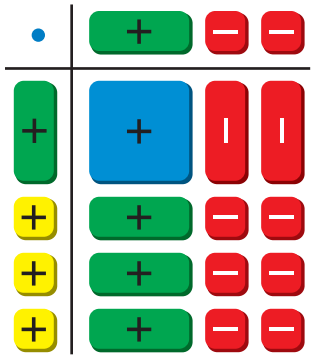
REASONING ABSTRACTLY

To be proficient in math, you need to reason abstractly and quantitatively. You need to pause as needed to recall the meanings of the symbols, operations, and quantities involved.

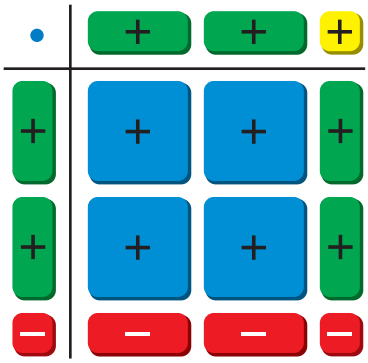
EXPLORATION 2 Multiplying Binomials Using Algebra Tiles

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles. In parts (c) and (d), first draw the rectangular array of algebra tiles that models each product.

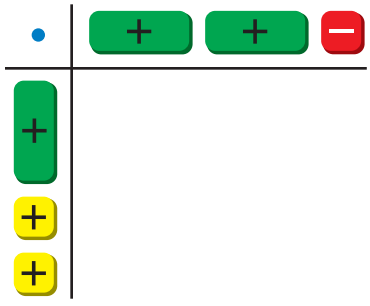
a. $(x + 3)(x - 2) =$ ____



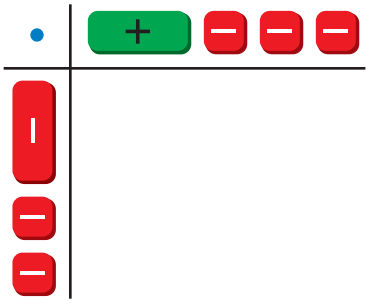
b. $(2x - 1)(2x + 1) =$ ____



c. $(x + 2)(2x - 1) =$ ____



d. $(-x - 2)(x - 3) =$ ____



Communicate Your Answer

3. How can you multiply two polynomials?

4. Give another example of multiplying two binomials using algebra tiles that is similar to those in Exploration 2.

7.2 Lesson

Core Vocabulary

FOIL Method, *p. 367*

Previous

polynomial
closed
binomial
trinomial

What You Will Learn

- Multiply binomials.
- Use the FOIL Method.
- Multiply binomials and trinomials.

Multiplying Binomials

The product of two polynomials is always a polynomial. So, like the set of integers, the set of polynomials is closed under multiplication. You can use the Distributive Property to multiply two binomials.

EXAMPLE 1 Multiplying Binomials Using the Distributive Property

Find (a) $(x + 2)(x + 5)$ and (b) $(x + 3)(x - 4)$.

SOLUTION

a. Use the horizontal method.

$(x + 2)(x + 5) = x(x + 5) + 2(x + 5)$ — Distribute $(x + 5)$ to each term of $(x + 2)$.

$= x(x) + x(5) + 2(x) + 2(5)$ — Distributive Property

$= x^2 + 5x + 2x + 10$ — Multiply.

$= x^2 + 7x + 10$ — Combine like terms.

▶ The product is $x^2 + 7x + 10$.

b. Use the vertical method.

$$\begin{array}{rl} & x + 3 \\ \times & x - 4 \\ \hline & -4x - 12 \\ & x^2 + 3x \\ \hline & x^2 - x - 12 \end{array}$$

Align like terms vertically.
Multiply $-4(x + 3)$. — Distributive Property
Multiply $x(x + 3)$. — Distributive Property
Combine like terms.

▶ The product is $x^2 - x - 12$.

EXAMPLE 2 Multiplying Binomials Using a Table

Find $(2x - 3)(x + 5)$.

SOLUTION

Step 1 Write each binomial as a sum of terms.

$(2x - 3)(x + 5) = [2x + (-3)](x + 5)$

Step 2 Make a table of products.

	$2x$	-3
x	$2x^2$	$-3x$
5	$10x$	-15

▶ The product is $2x^2 - 3x + 10x - 15$, or $2x^2 + 7x - 15$.

Monitoring Progress

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Use the Distributive Property to find the product.

1. $(y + 4)(y + 1)$

2. $(z - 2)(z + 6)$

Use a table to find the product.

3. $(p + 3)(p - 8)$

4. $(r - 5)(2r - 1)$

Using the FOIL Method

The **FOIL Method** is a shortcut for multiplying two binomials.

Core Concept

FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms, $(x + 1)(x + 2)$ → $x(x) = x^2$

Outer terms, $(x + 1)(x + 2)$ → $x(2) = 2x$

Inner terms, and $(x + 1)(x + 2)$ → $1(x) = x$

Last terms. $(x + 1)(x + 2)$ → $1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

EXAMPLE 3 Multiplying Binomials Using the FOIL Method

Find each product.

a. $(x - 3)(x - 6)$ **b.** $(2x + 1)(3x - 5)$

SOLUTION

a. Use the FOIL Method.

$$\begin{aligned} (x - 3)(x - 6) &= \underset{\text{First}}{x(x)} + \underset{\text{Outer}}{x(-6)} + \underset{\text{Inner}}{(-3)(x)} + \underset{\text{Last}}{(-3)(-6)} && \text{FOIL Method} \\ &= x^2 + (-6x) + (-3x) + 18 && \text{Multiply.} \\ &= x^2 - 9x + 18 && \text{Combine like terms.} \end{aligned}$$

▶ The product is $x^2 - 9x + 18$.

b. Use the FOIL Method.

$$\begin{aligned} (2x + 1)(3x - 5) &= \underset{\text{First}}{2x(3x)} + \underset{\text{Outer}}{2x(-5)} + \underset{\text{Inner}}{1(3x)} + \underset{\text{Last}}{1(-5)} && \text{FOIL Method} \\ &= 6x^2 + (-10x) + 3x + (-5) && \text{Multiply.} \\ &= 6x^2 - 7x - 5 && \text{Combine like terms.} \end{aligned}$$

▶ The product is $6x^2 - 7x - 5$.

Monitoring Progress

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Use the FOIL Method to find the product.

5. $(m - 3)(m - 7)$ **6.** $(x - 4)(x + 2)$

7. $\left(2u + \frac{1}{2}\right)\left(u - \frac{3}{2}\right)$ **8.** $(n + 2)(n^2 + 3)$

Multiplying Binomials and Trinomials

EXAMPLE 4 Multiplying a Binomial and a Trinomial

Find $(x + 5)(x^2 - 3x - 2)$.

SOLUTION

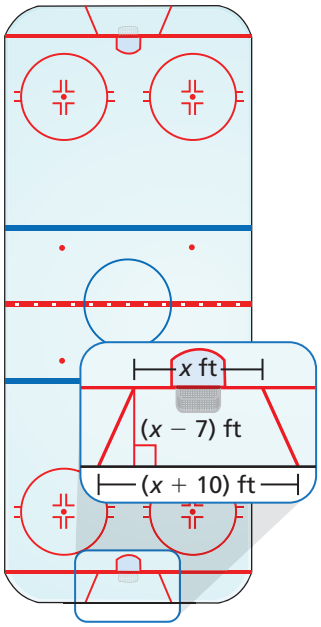
$$\begin{array}{rll} & x^2 - 3x - 2 & \text{Align like terms vertically.} \\ \times & x + 5 & \\ \hline & 5x^2 - 15x - 10 & \text{Distributive Property} \\ & x^3 - 3x^2 - 2x & \text{Distributive Property} \\ \hline & x^3 + 2x^2 - 17x - 10 & \text{Combine like terms.} \end{array}$$

Multiply $5(x^2 - 3x - 2)$.

Multiply $x(x^2 - 3x - 2)$.

▶ The product is $x^3 + 2x^2 - 17x - 10$.

EXAMPLE 5 Solving a Real-Life Problem



In hockey, a goalie behind the goal line can only play a puck in the trapezoidal region.

a. Write a polynomial that represents the area of the trapezoidal region.

b. Find the area of the trapezoidal region when the shorter base is 18 feet.

SOLUTION

a. $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(x - 7)[x + (x + 10)]$ — Substitute.

$= \frac{1}{2}(x - 7)(2x + 10)$ — Combine like terms.

F O I L

$= \frac{1}{2}[2x^2 + 10x + (-14x) + (-70)]$ — FOIL Method

$= \frac{1}{2}(2x^2 - 4x - 70)$ — Combine like terms.

$= x^2 - 2x - 35$ — Distributive Property

▶ A polynomial that represents the area of the trapezoidal region is $x^2 - 2x - 35$.

b. Find the value of $x^2 - 2x - 35$ when $x = 18$.

$x^2 - 2x - 35 = 18^2 - 2(18) - 35$ — Substitute 18 for x.

$= 324 - 36 - 35$ — Simplify.

$= 253$ — Subtract.

▶ The area of the trapezoidal region is 253 square feet.

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Find the product.

9. $(x + 1)(x^2 + 5x + 8)$

10. $(n - 3)(n^2 - 2n + 4)$

11. WHAT IF? In Example 5(a), how does the polynomial change when the longer base is extended by 1 foot? Explain.

7.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Describe two ways to find the product of two binomials.

2. **WRITING** Explain how the letters of the word FOIL can help you to remember how to multiply two binomials.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the Distributive Property to find the product. *(See Example 1.)*

3. $(x + 1)(x + 3)$
4. $(y + 6)(y + 4)$
5. $(z - 5)(z + 3)$
6. $(a + 8)(a - 3)$
7. $(g - 7)(g - 2)$
8. $(n - 6)(n - 4)$
9. $(3m + 1)(m + 9)$
10. $(5s + 6)(s - 2)$

In Exercises 11–18, use a table to find the product. *(See Example 2.)*

11. $(x + 3)(x + 2)$
12. $(y + 10)(y - 5)$
13. $(h - 8)(h - 9)$
14. $(c - 6)(c - 5)$
15. $(3k - 1)(4k + 9)$
16. $(5g + 3)(g + 8)$
17. $(-3 + 2j)(4j - 7)$
18. $(5d - 12)(-7 + 3d)$

ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in finding the product of the binomials.**

19. ✗

$$(t - 2)(t + 5) = t - 2(t + 5)$$
$$= t - 2t - 10$$
$$= -t - 10$$

20. ✗

$(x - 5)(3x + 1)$

	$3x$	1
x	$3x^2$	x
5	$15x$	5

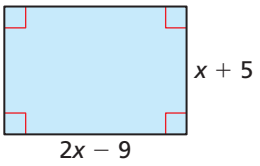
$(x - 5)(3x + 1) = 3x^2 + 16x + 5$

In Exercises 21–30, use the FOIL Method to find the product. *(See Example 3.)*

21. $(b + 3)(b + 7)$
22. $(w + 9)(w + 6)$
23. $(k + 5)(k - 1)$
24. $(x - 4)(x + 8)$
25. $\left(q - \frac{3}{4}\right)\left(q + \frac{1}{4}\right)$
26. $\left(z - \frac{5}{3}\right)\left(z - \frac{2}{3}\right)$
27. $(9 - r)(2 - 3r)$
28. $(8 - 4x)(2x + 6)$
29. $(w + 5)(w^2 + 3w)$
30. $(v - 3)(v^2 + 8v)$

MATHEMATICAL CONNECTIONS **In Exercises 31–34, write a polynomial that represents the area of the shaded region.**

31.



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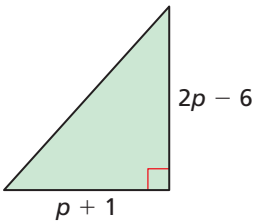

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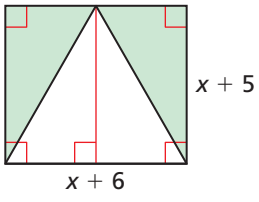

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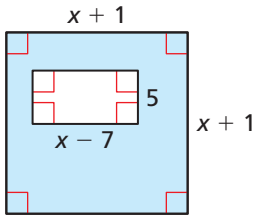
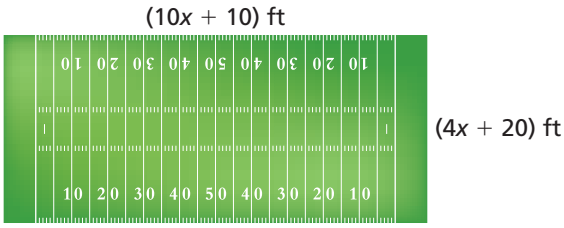


In Exercises 35–42, find the product. *(See Example 4.)*

35. $(x + 4)(x^2 + 3x + 2)$
36. $(f + 1)(f^2 + 4f + 8)$
37. $(y + 3)(y^2 + 8y - 2)$
38. $(t - 2)(t^2 - 5t + 1)$
39. $(4 - b)(5b^2 + 5b - 4)$
40. $(d + 6)(2d^2 - d + 7)$
41. $(3e^2 - 5e + 7)(6e + 1)$
42. $(6v^2 + 2v - 9)(4 - 5v)$

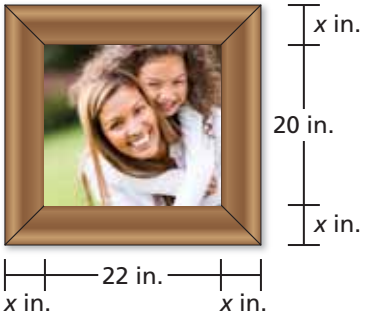
43. MODELING WITH MATHEMATICS The football field is rectangular. *(See Example 5.)*



a. Write a polynomial that represents the area of the football field.

b. Find the area of the football field when the width is 160 feet.

44. MODELING WITH MATHEMATICS You design a frame to surround a rectangular photo. The width of the frame is the same on every side, as shown.



a. Write a polynomial that represents the combined area of the photo and the frame.

b. Find the combined area of the photo and the frame when the width of the frame is 4 inches.

45. WRITING When multiplying two binomials, explain how the degree of the product is related to the degree of each binomial.

46. THOUGHT PROVOKING Write two polynomials that are not monomials whose product is a trinomial of degree 3.

47. MAKING AN ARGUMENT Your friend says the FOIL Method can be used to multiply two trinomials. Is your friend correct? Explain your reasoning.

48. HOW DO YOU SEE IT? The table shows one method of finding the product of two binomials.

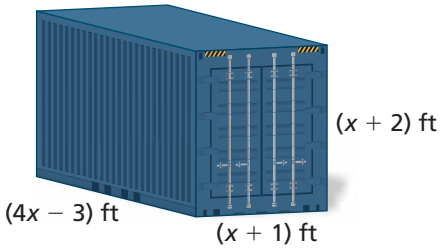
	$-4x$	3
$-8x$	a	b
-9	c	d

a. Write the two binomials being multiplied.

b. Determine whether a, b, c, and d will be positive or negative when $x > 0$.

49. COMPARING METHODS You use the Distributive Property to multiply $(x + 3)(x - 5)$. Your friend uses the FOIL Method to multiply $(x - 5)(x + 3)$. Should your answers be equivalent? Justify your answer.

50. USING STRUCTURE The shipping container is a rectangular prism. Write a polynomial that represents the volume of the container.



51. ABSTRACT REASONING The product of $(x + m)(x + n)$ is $x^2 + bx + c$.

a. What do you know about m and n when $c > 0$?

b. What do you know about m and n when $c < 0$?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the absolute value function as a piecewise function. *(Section 4.7)*

52. $y = |x| + 4$

53. $y = 6|x - 3|$

54. $y = -4|x + 2|$

Simplify the expression. Write your answer using only positive exponents. *(Section 6.1)*

55. $10^2 \cdot 10^9$

56. $\dfrac{x^5 \cdot x}{x^8}$

57. $(3z^6)^{-3}$

58. $\left(\dfrac{2y^4}{y^3}\right)^{-2}$